Many Holes in Hindley-Milner

Sam Lindley

Laboratory for Foundations of Computer Science,
University of Edinburgh

Sam.Lindley@ed.ac.uk

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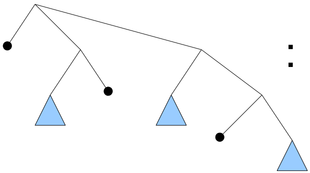
Plugging many-holed contexts



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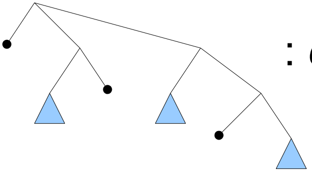

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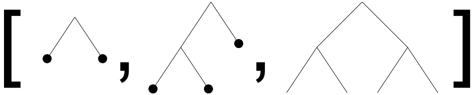

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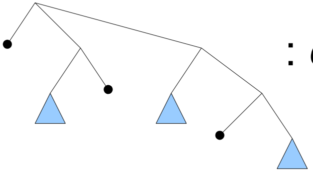

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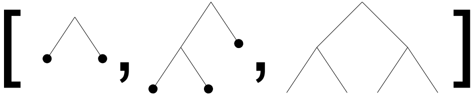

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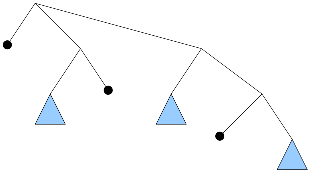

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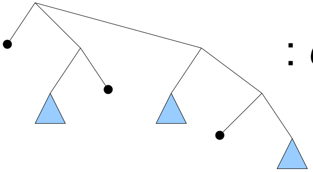

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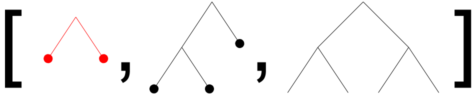

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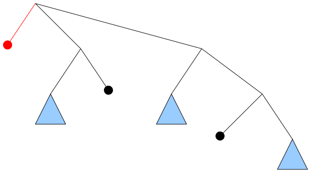

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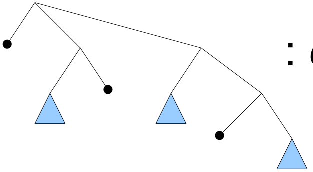

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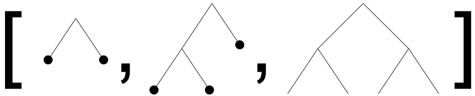

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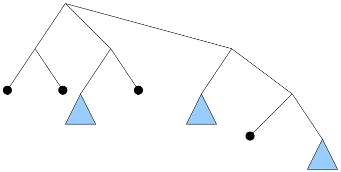

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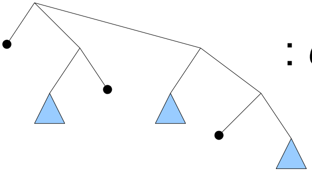

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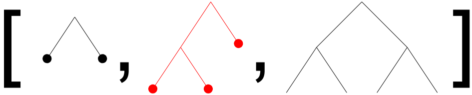

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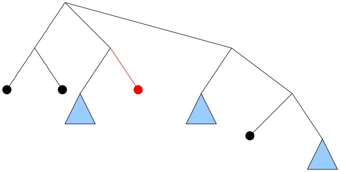

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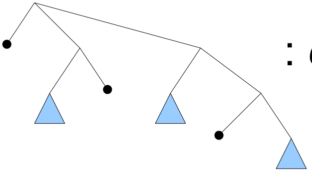

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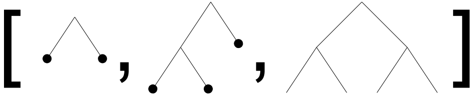

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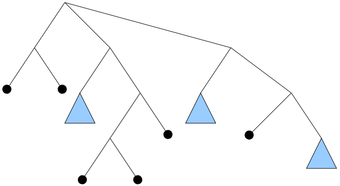

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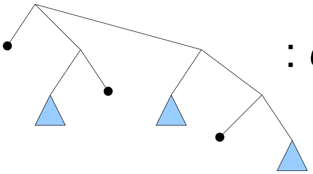

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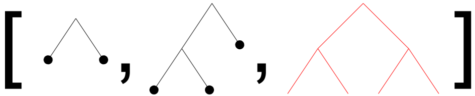

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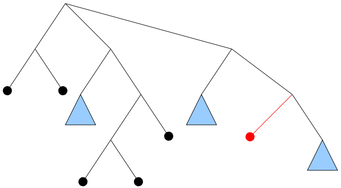

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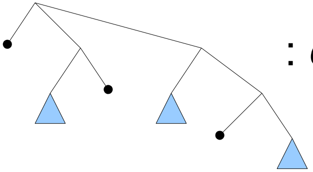

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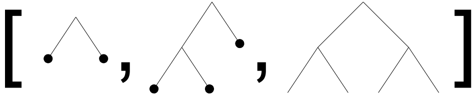

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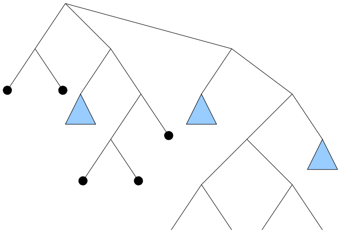

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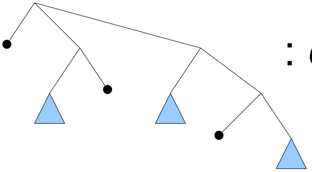

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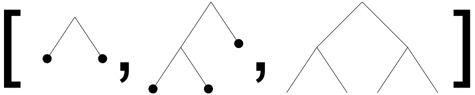

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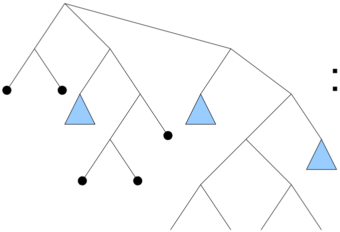

: *context*(5)

Lists of length n

Represent the naturals as type-level Peano numbers.

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type 'n s    (* successor *)
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```
type z       (* zero *)
type 'n s    (* successor *)
```

Represent the length of a list as a phantom type parameter.

```
module SimpleNList : sig
  type ('length, 'elem_type) t

  val nil : (z, 'a) t
  val cons : 'a × ('n, 'a) t → ('n s, 'a) t
end = struct
  type ('n, 'a) t = 'a list

  let nil = []
  let cons (x, xs) = x :: xs
end
```

Can we type list append?

From (Xi, JFP 2007)

> *A correct implementation of the append function on lists should return a list of length* $m + n$ *when given two lists of length* m *and* n*, respectively. This property, however, cannot be captured by the type system of ML, and the inadequacy can be remedied if we introduce a restricted form of dependent types.*

Difference types: type-level addition in ML

Idea: represent naturals as pairs denoting the difference between two Peano numbers, then addition becomes composition: $(m, n) + (l, m) = (l, n)$.

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```
module Nat : sig                                      struct
  type 'i t                                             type 'i t = int
  val zero : ('m×'m) t                                  let zero = 0
  val succ : ('m×'n) t → ('m×'n s) t                    let succ n = n+1
  val add : ('m×'n) t × ('l×'m) t → ('l×'n) t           let add (n,m) = n+m
  val to_int : 'i t → int                               let to_int n = n
end =                                                 end
```

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  val to_int : 'i t → int
end =
```

```
struct
  type 'i t = int
  let zero = 0
  let succ n = n+1
  let add (n,m) = n+m
  let to_int n = n
end
```

Problem: syntactic non-values cannot be polymorphic.

succ zero : ('_n×'_n s) *Nat.t*

add (*succ* (*succ zero*), *succ* (*succ zero*)) : ('_n×'_n s s s s) *Nat.t*

The relaxed value restriction

Any free type variables which occur only in *covariant positions* outside of reference types can be safely generalised, even for terms that are not syntactic values (Garrigue, FLOPS 2004).

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```

```
module Nat : sig                                        struct
  type +'i t                                              type 'i t = int
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  val add : ('m×'n) t × ('l×'m) t → ('l×'n) t             let add (n,m) = n+m
  val to_int : 'i t → int                                 let to_int n = n
end =                                                   end
```

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```
module Nat : sig                                          struct
  type +'i t                                                type 'i t = int
  val zero : ('m×'m) t                                      let zero = 0
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  val add : ('m×'n) t × ('l×'m) t → ('l×'n) t               let add (n,m) = n+m
  val to_int : 'i t → int                                   let to_int n = n
end =                                                     end
```

The covariance annotations enable generalisation!

succ zero : ('n×'n *s*) *Nat.t*

add (*succ* (*succ zero*), *succ* (*succ zero*)) : ('n×'n *s s s s*) *Nat.t*

Appending lists of length n

```
module NList : sig
  type (+'length, +'elem_type) t

  val nil : ('m×'m, 'a) t
  val cons : 'a × ('m×'n, 'a) t → ('m×'n s, 'a) t
  val append : ('m×'n, 'a) t × ('l×'m, 'a) t → ('l×'n, 'a) t

  val to_list : ('i, 'a) t → 'a list
end = struct
  type ('i, 'a) t = 'a list

  let nil = []
  let cons (x, xs) = x :: xs
  let append (xs, ys) = xs @ ys

  let to_list xs = xs
end
```

An XML context datatype

$$
\begin{array}{l}
\text{type } \mathit{xml} = \\
\quad \mid \mathit{Empty} \\
\quad \mid \mathit{Text} \text{ of } \mathit{string} \\
\quad \mid \mathit{Tag} \text{ of } \mathit{string} \times \mathit{xml} \\
\quad \mid \mathit{Concat} \text{ of } \mathit{xml} \times \mathit{xml} \\
\quad \mid \mathit{Hole}
\end{array}
$$

An XML context datatype

type *xml* =
  | *Empty*
  | *Text* of *string*
  | *Tag* of *string* × *xml*
  | *Concat* of *xml* × *xml*
  | *Hole*

Dynamically checked plugging operation

$$dynamic\_plug : xml \times xml\ list \rightarrow xml$$

Counting holes

```
module NContext : sig
  type +'holes t

  val empty : ('m×'m) t
  val text : string → ('m×'m) t
  val tag : string × 'i t → 'i t
  val concat : ('m×'n) t × ('l×'m) t → ('l×'n) t
  val hole : ('n×'n s) t

  val to_xml : 'i t → xml
end = struct
  type 'i t = xml

  let empty = Empty
  let text s = Text s
  let tag (s, x) = Tag (s, x)
  let concat (x, y) = Concat (x, y)
  let hole = Hole

  let to_xml k = k
end
```

Lists of contexts

We want to be able to plug a heterogeneous list of many-holed contexts (each context in the list may have a different number of holes) into a many-holed context.

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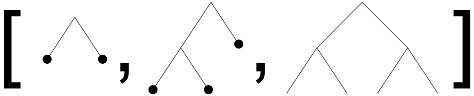

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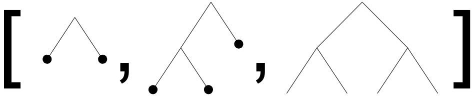


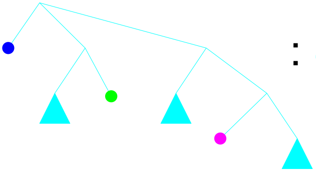
: *contextList*(5,3)

We don't need to! We only need to know the total number of holes and the total length of the list.

Plugging

```
module NContext : sig
  ...
  type (+'holes, +'length) ts

  val nil : ('p×'p, 'm×'m) ts
  val cons : ('p×'q) t × ('o×'p, 'm×'n) ts → ('o×'q, 'm×'n s) ts
  val append : ('p×'q, 'm×'n) ts × ('o×'p, 'l×'m) ts → ('o×'q, 'l×'n) ts

  val plug : 'j t × ('i, 'j) ts → 'i t
end = struct
  ...
  type ('i, 'j) ts = ('j, xml) NList.t

  let nil = NList.nil
  let cons (x, xs) = NList.cons (to_xml x, xs)
  let append = NList.append

  let plug (k, xs) = dynamic_plug (k, NList.to_list xs)
end
```

Marking holes and contexts



: *context*(⋮)

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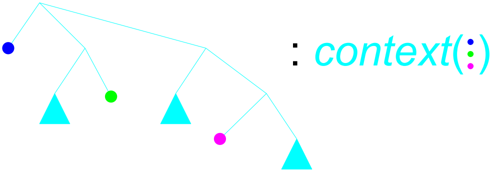

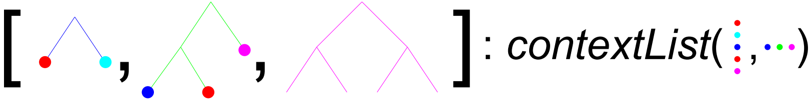

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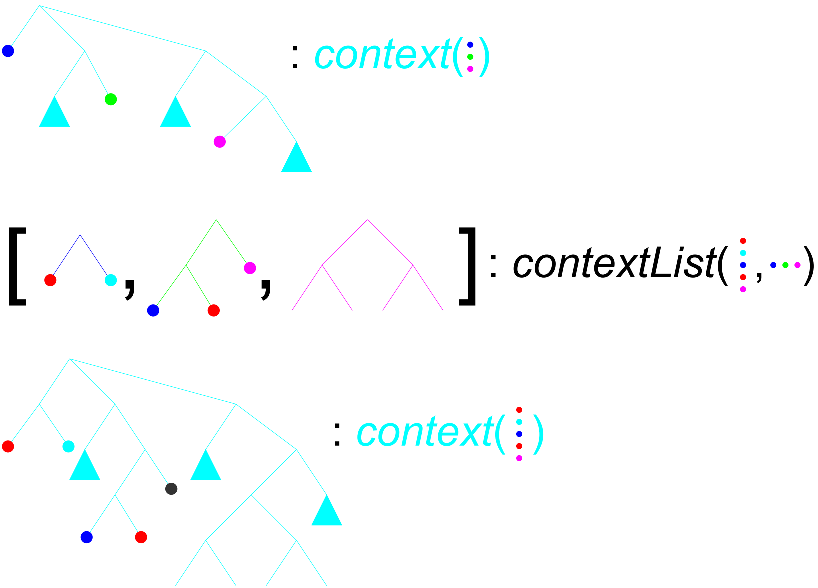

MiniXHTML

A small subset of XHTML introduced by Elsman and Larsen (PADL 2004) to demonstrate static typing of XHTML using phantom types.

```
<!ENTITY %block "p|table|pre">
<!ENTITY %inline "%inpre|big">
<!ENTITY %flow "%block|%inline">
<!ENTITY %inpre "#PCDATA|em">
<!ENTITY %td "td">
<!ENTITY %tr "tr">

<!ELEMENT p (%inline)*>
<!ELEMENT em (%inline)*>
<!ELEMENT big (%inline)*>
<!ELEMENT pre (%inpre)*>
<!ELEMENT td (%flow)*>
<!ELEMENT tr (%td)+>
<!ELEMENT table (%tr)+>
```

Statically typed MiniXHTML contexts

```
module MX : sig
  type (+'blk, +'inl) flw and tr and td
  type blk and inl and no and inpre
  type preclosed

  type (+'holes, +'mark) t

  val empty : ('m×'m, 'h) t
  val text : string → ('m×'m, 'h) t
  val p : ('i, (no,inl)flw×'c) t → ('i, (blk,'b)flw×'c) t
  val em : ('i, (no,inl)flw×'c) t → ('i, ('b,inl)flw×'c) t
  val pre : ('i, (no,inl)flw×inpre) t → ('i, (blk,'b)flw×'c) t
  val big : ('i, (no,inl)flw×'c) t → ('i, ('b,inl)flw×preclosed) t
  val table : ('i, tr×'c) t → ('i, (blk,'b)flw×'c) t
  val tr : ('i, td×'c) t → ('i, tr×'c) t
  val td : ('i, (blk,inl)flw×'c) t → ('i, td×'c) t
  val concat : ('m×'n, 'h) t × ('l×'m, 'h) t → ('l×'n, 'h) t
  val hole : ('m×('m×'h) s, 'h) t

  val to_xml : ('i, 'h) t → xml
end = ...
```

Statically typed MiniXHTML contexts (continued)

```
module MX : sig
  ...
  type (+'holes, +'length) ts

  val nil : ('m×'m, 'n×'n) ts
  val cons : ('p×'q, 'h) t × ('o×'p, 'm×'n) ts
                  → ('o×'q, 'm×(('n×'h) s)) ts
  val append : ('p×'q, 'm×'n) ts × ('o×'p, 'l×'m) ts → ('o×'q, 'l×'n) ts

  val plug : ('j, 'h) t × ('i, 'j) ts → ('i, 'h) t
end = ...
```

Related work

Encoding types

- Type-indexed families of functions in ML (Danvy, JFP 1998; Yang, ICFP 1998; Fridlender and Indrika, JFP 2000; ...)
- "Faking" dependent types using Haskell type classes (McBride, JFP 2002)

Difference types

- Polymorphic record concatenation for free (Rémy, POPL 1992)
- Hughes lists (Hughes, Information processing letters 1986)
- Prolog difference lists

Original motivation: *formlets*

- The essence of form abstraction (Cooper et al, APLAS 2008).
- Support syntactic sugar that allows form bindings to be embedded inside HTML.
- Basic desugaring uses a *tag* operation to build up the HTML presentation around a formlet *a single element at a time*.

  $$\mathit{tag} : \mathit{string} \rightarrow \mathit{attrs} \rightarrow \alpha\ \mathit{formlet} \rightarrow \alpha\ \mathit{formlet}$$

  - Easy to type, but the HTML and the part of the formlet abstraction that handles form input are intermingled.
- Alternative desugaring: replace *tag* with a many-holed *plug* operation.
  - Provides a clean separation between the HTML and the handling code. Relies on this work to statically type the *plug* operation.

Conclusions

- With difference types:
  - we *can* type list append in ML;
  - we *can* type many-holed plugging in ML;
  - we *can* mark holes and contexts with extra type information;
  - the relaxed value restriction helps a lot;
  - error messages are verbose.
- Type classes (GHC), indexed types (DML) and GADTs (GHC) are all more expressive than difference types.
  - In particular, they all provide better support for non-trivial destructors.