Soundness is not Sufficient

Fritz Henglein
DIKU

Shonan Village, 2011-09-25

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- Ask, comment, interrupt any time.

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- Propose technical criteria for capturing some aspects of the informal criteria
- Identify questions for further work, both conceptual and technical.

Program property

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- A program property P is **semantic** (**extensional**) if

$$\mathbf{p \cong q \Rightarrow (P(p) \Leftrightarrow P(q))}$$

- A program property P is **trivial** if

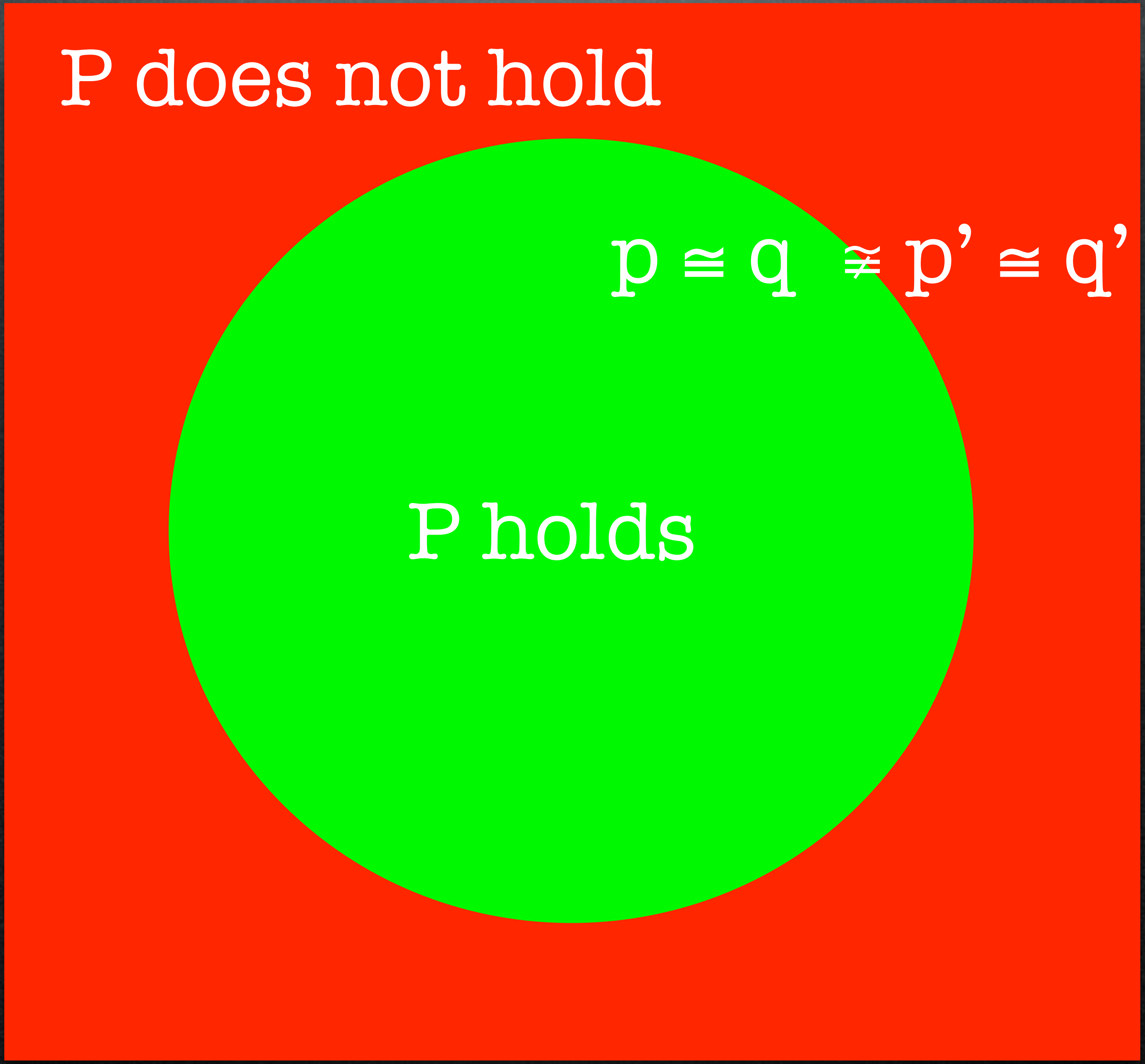
  P(p) for all p, or ¬P(p) for all p.

Rice’s Curse

Theorem:
Let L be a Turing-complete programming language, P a nontrivial semantic program property.
Then P is undecidable.

Rice, Classes of recursively enumerable sets and their decision problems, Trans. AMS 1953

Rice’s Curse, pictorially



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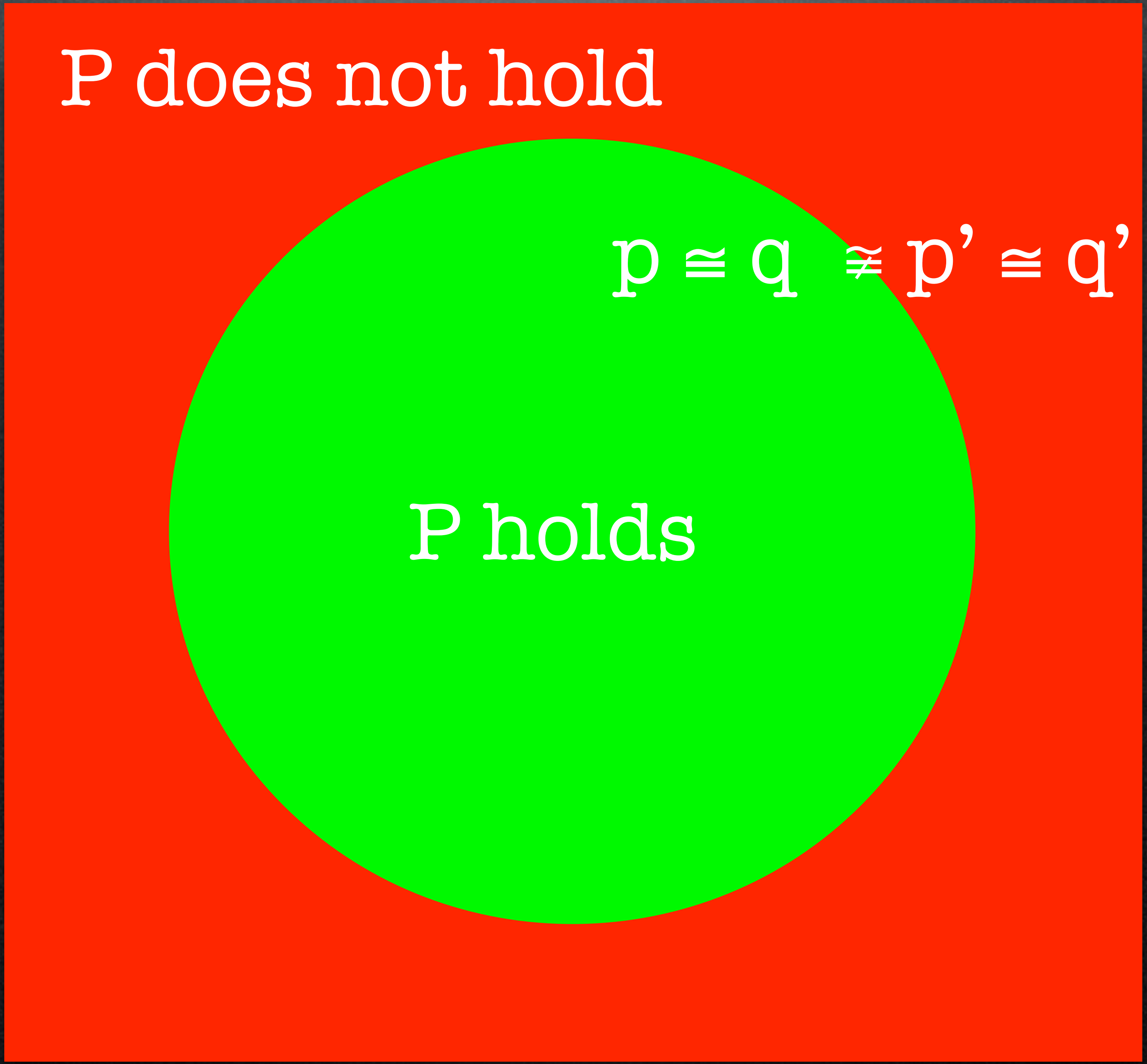


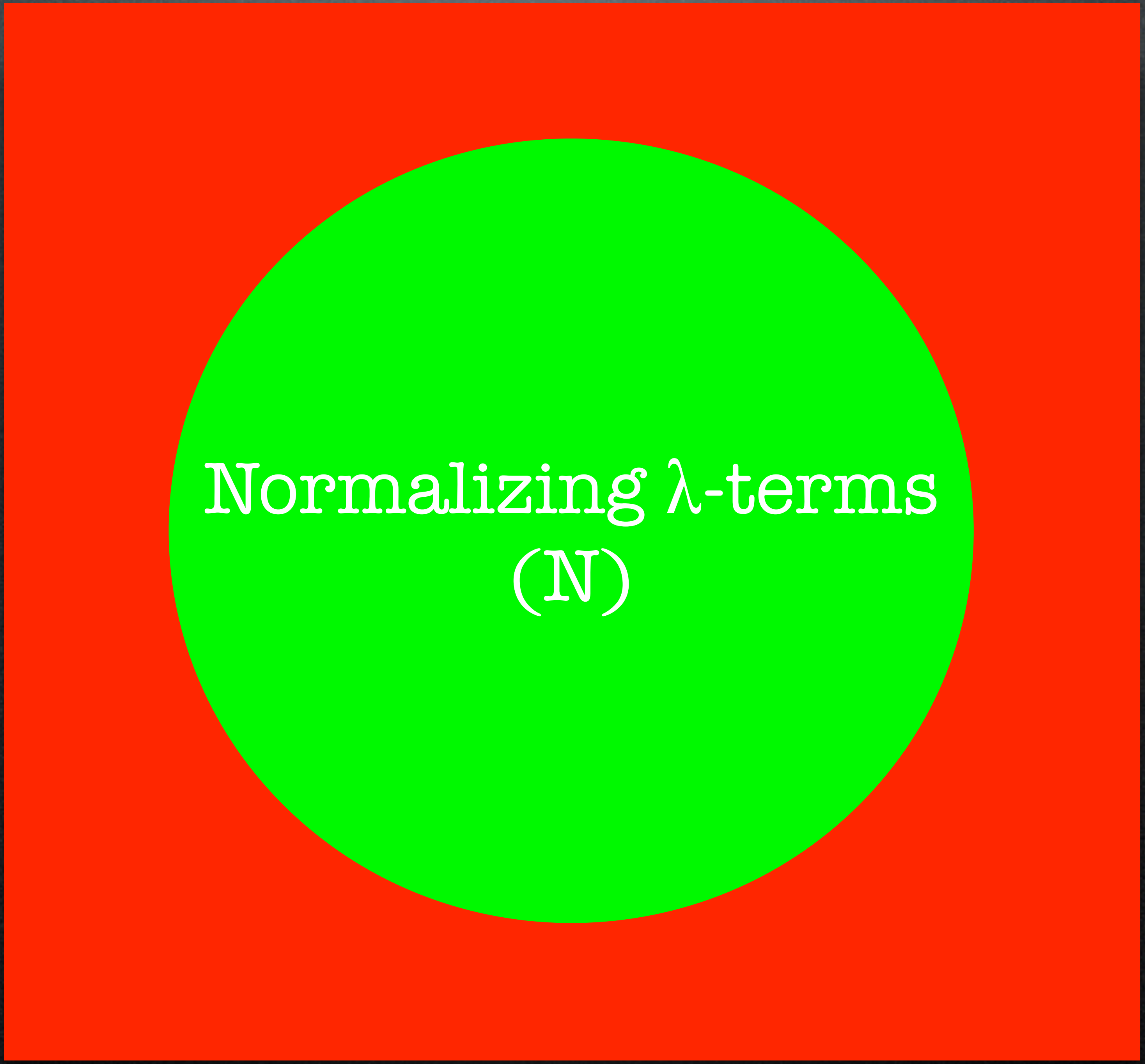
P does not hold

$p \cong q \ncong p' \cong q'$

P holds

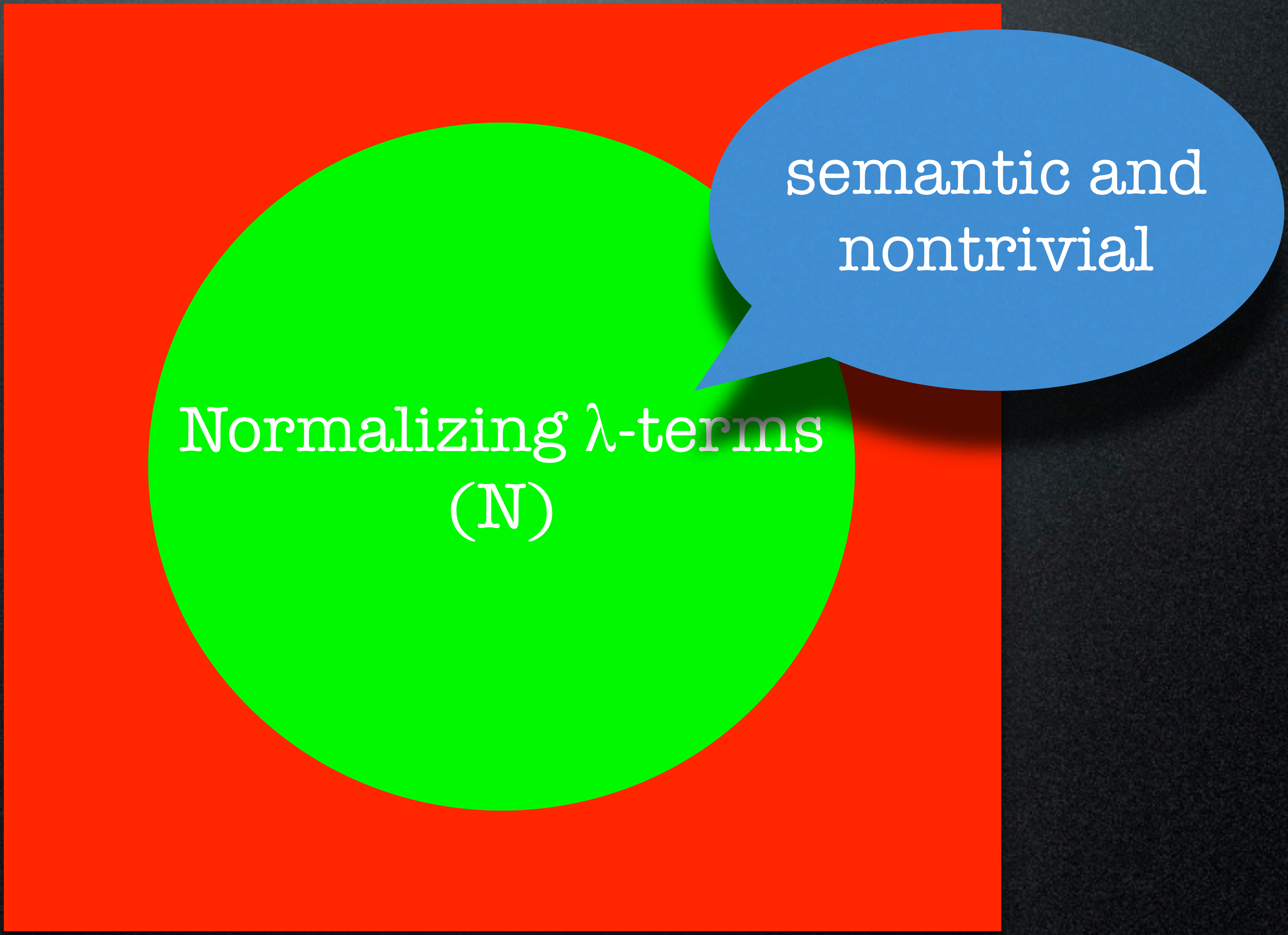
P is not decidable!

Rice’s Curse: Example



Normalizing λ-terms
(N)

Rice’s Curse: Example



semantic and nontrivial

Normalizing λ-terms (N)

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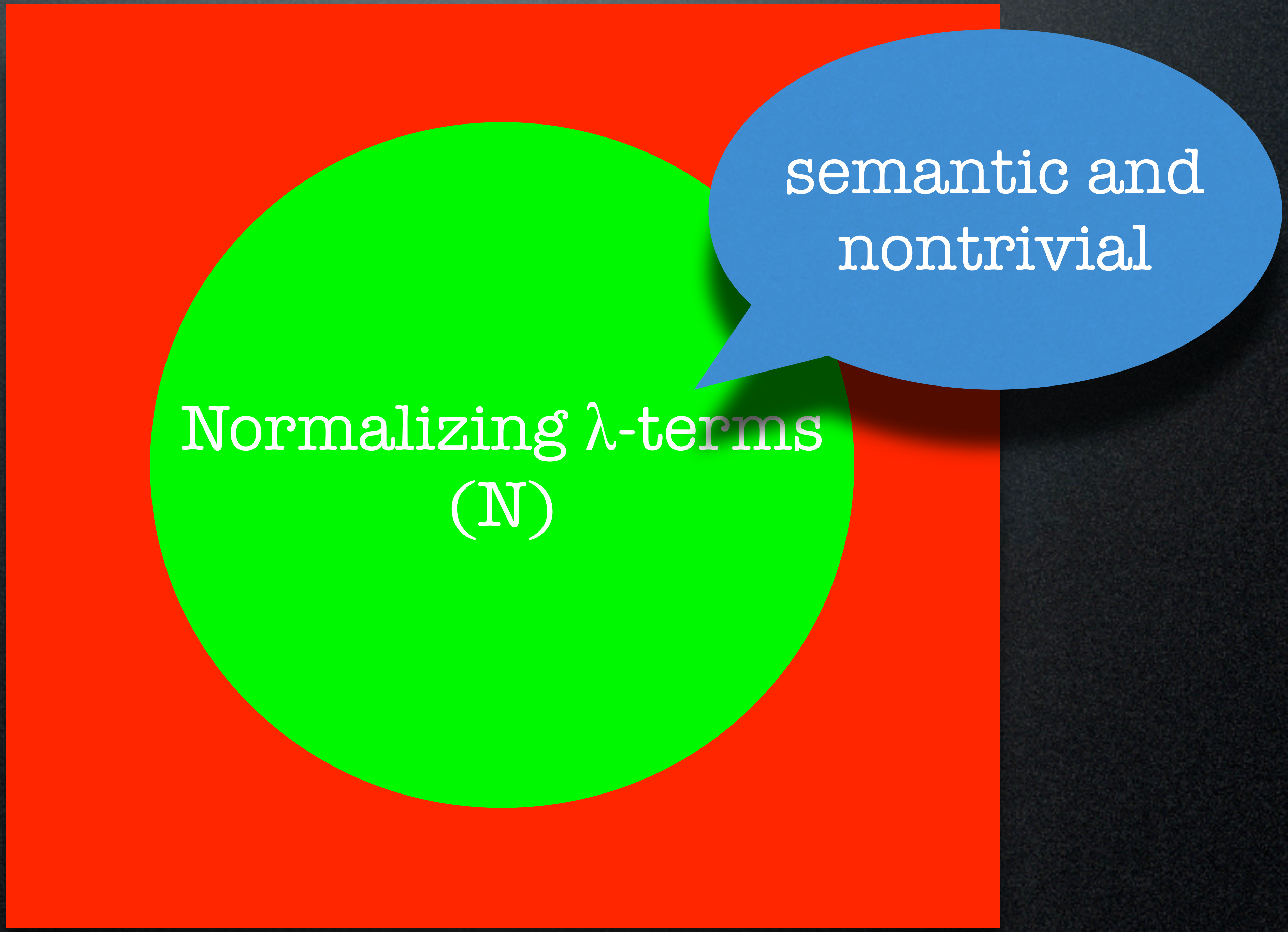

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Normalizing λ-terms (N)

Corollary: N is not decidable!

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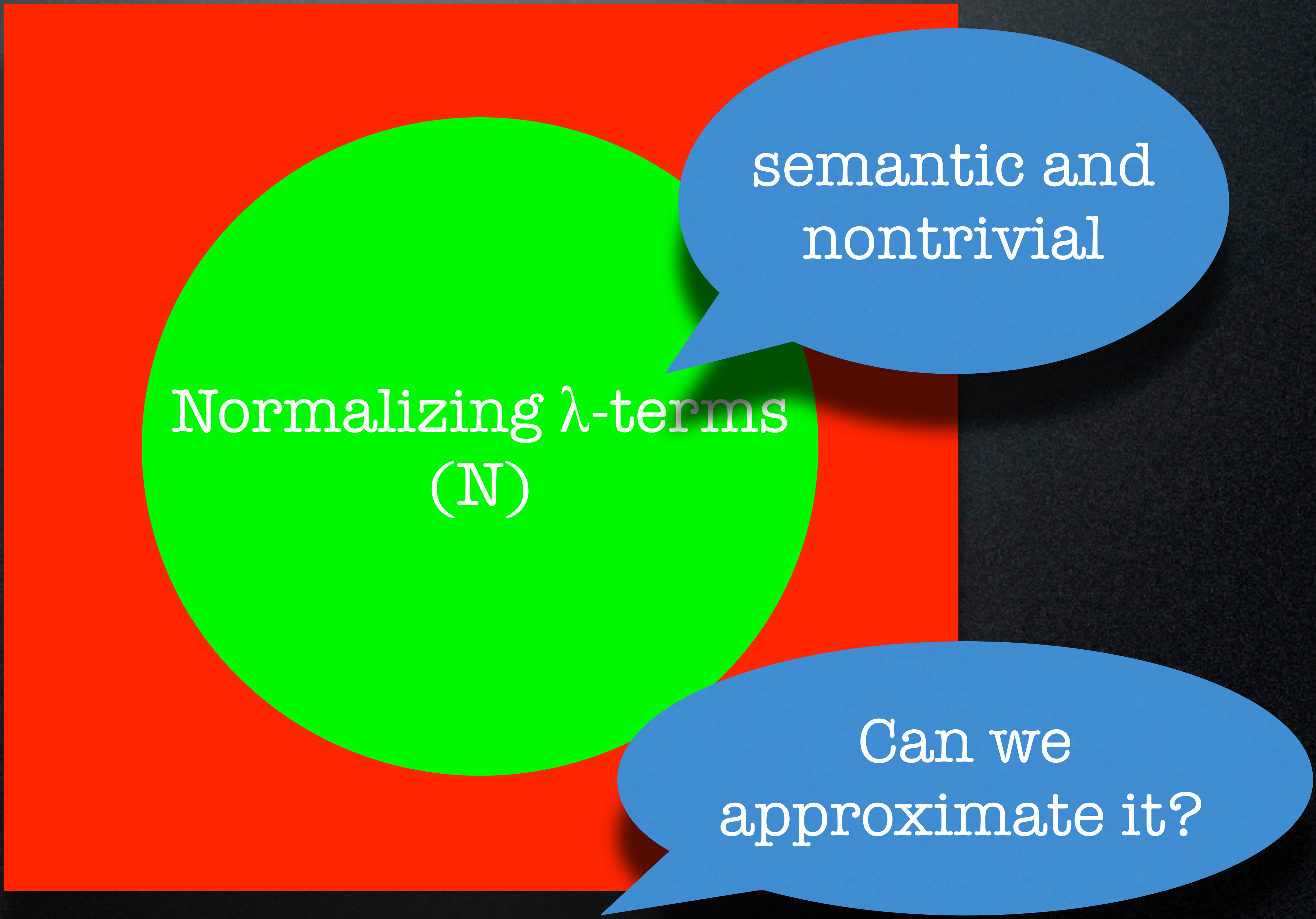

Normalizing λ-terms (N)

semantic and nontrivial

Can we approximate it?

Corollary: N is not decidable!

Static analysis

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- Given:

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    - P: Extensional program property

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    Is that sufficient? No, we also want...

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    - **Goodness**

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What does “good” mean??

Goodness characteristics

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- **Usefulness**:
  - Has some effective use

Goodness characteristics

- **Usefulness**:
  - Has some effective use
- **Declarative specification**:
  - Separation of **what** the analysis computes from **how** it computes it (the particular algorithm[s] used)

Goodness characteristics

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- **Unimprovability**:
  - Can’t get **better** approximation at **lower** computational cost

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- **Unimprovability**:
  - Can’t get **better** approximation at **lower** computational cost
- **Predictability**:
  - Predictability under program transformations

Goodness tics

Algorithm need not be compositional, only its result

- **Compositional certification**
  - Explicit, modular (syntax-oriented), efficiently checkable logical explanation of analysis results
- **Constructive interpretation**
  - Operational interpretation of certificate, not just of yes/no answer

Goodness characteristics

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- **Adaptiveness:**
  - Easy instances are handled efficiently
  - Hard instances may take more time.
- **Parameter sensitivity**
  - Scale well with parameter, which captures expectations on input distribution.

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Property of particular **algorithm** A implementing an analysis S

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(Not developed here)

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Static Analysis for N

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- Imagine we want to analyze N

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- Is System F typability a good static analysis for N?

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- Sound? ✔

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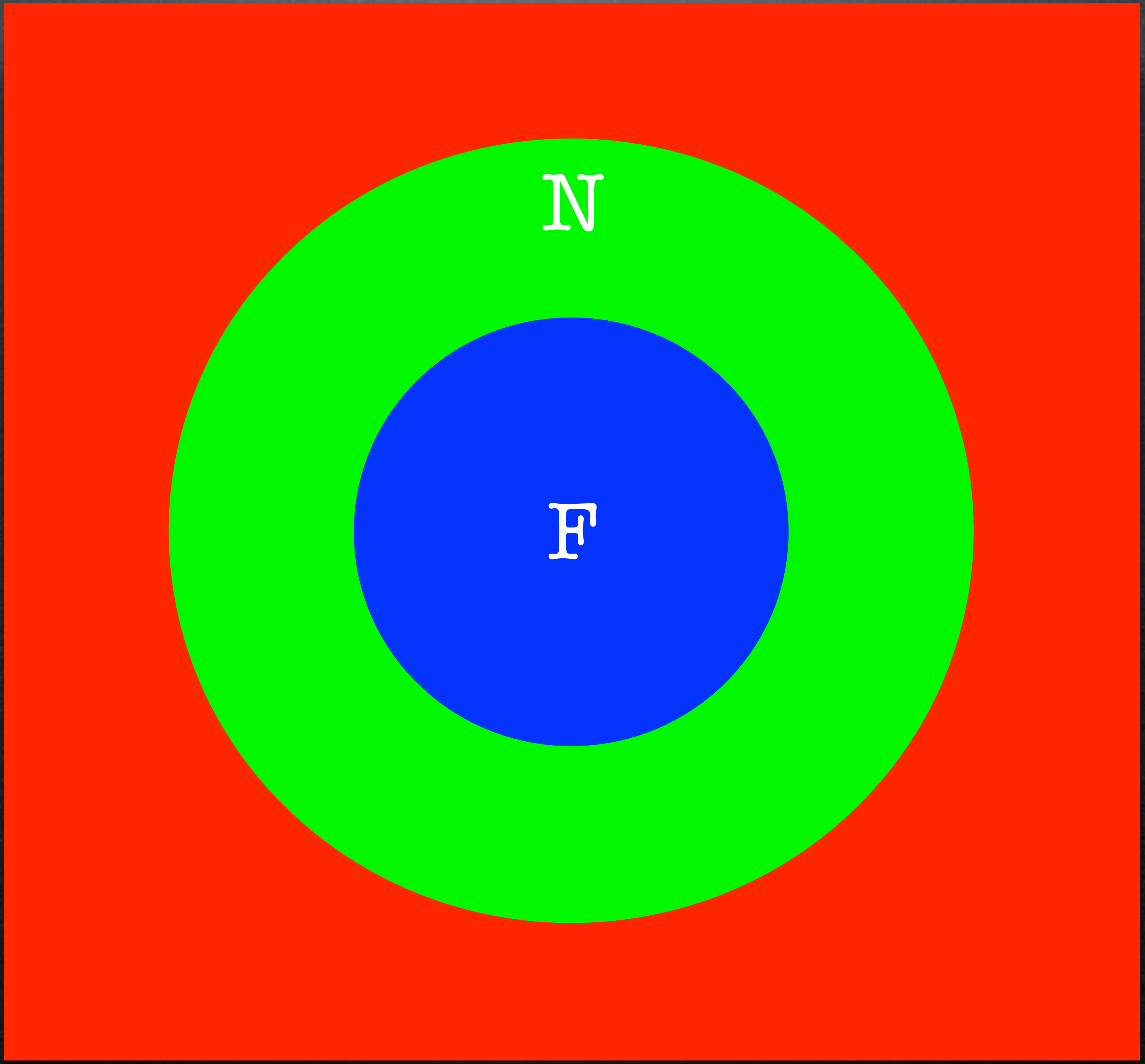
- Predictability properties? (✔)
- Unimprovability? Hmm...

Static Analysis for N



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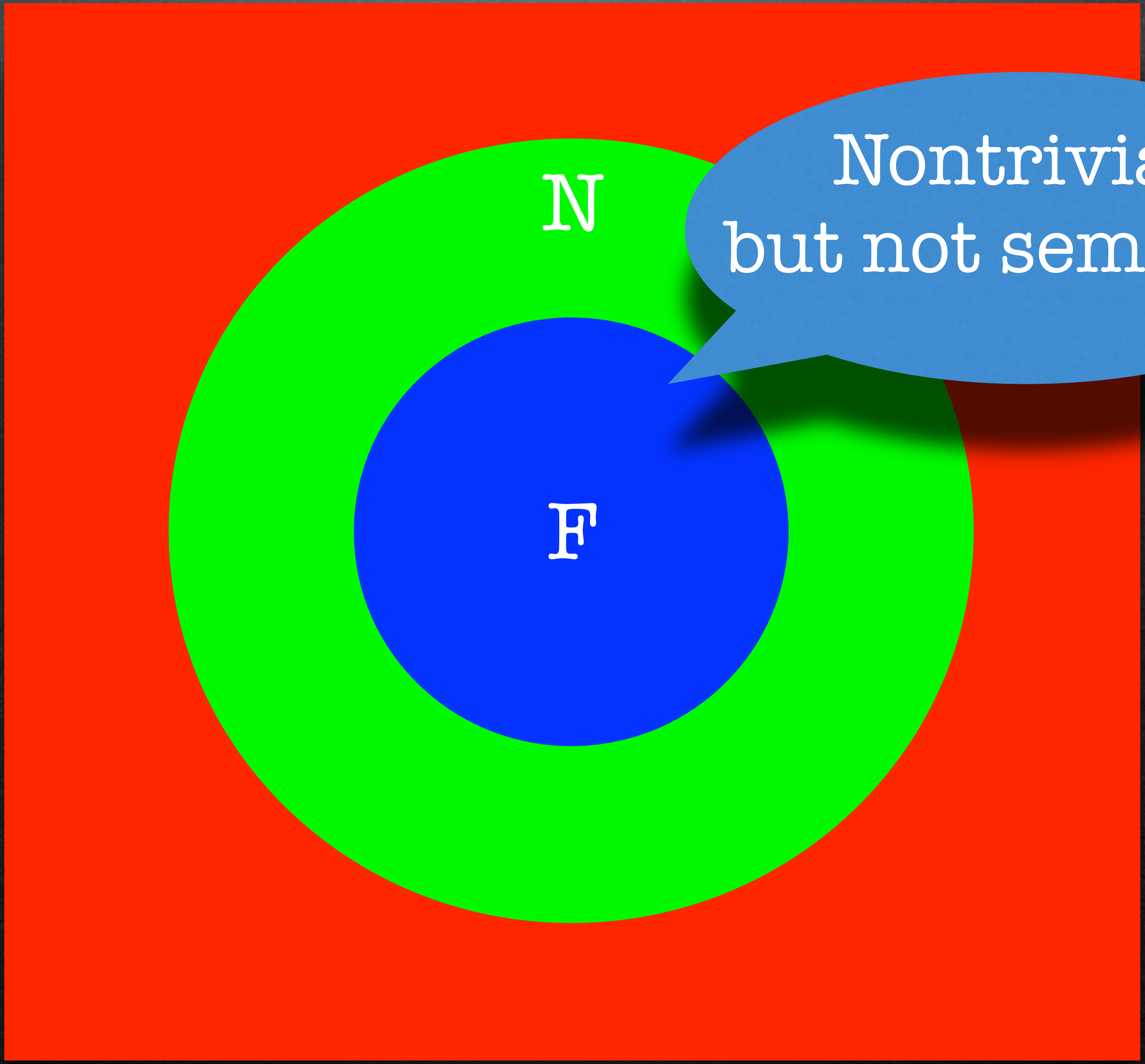
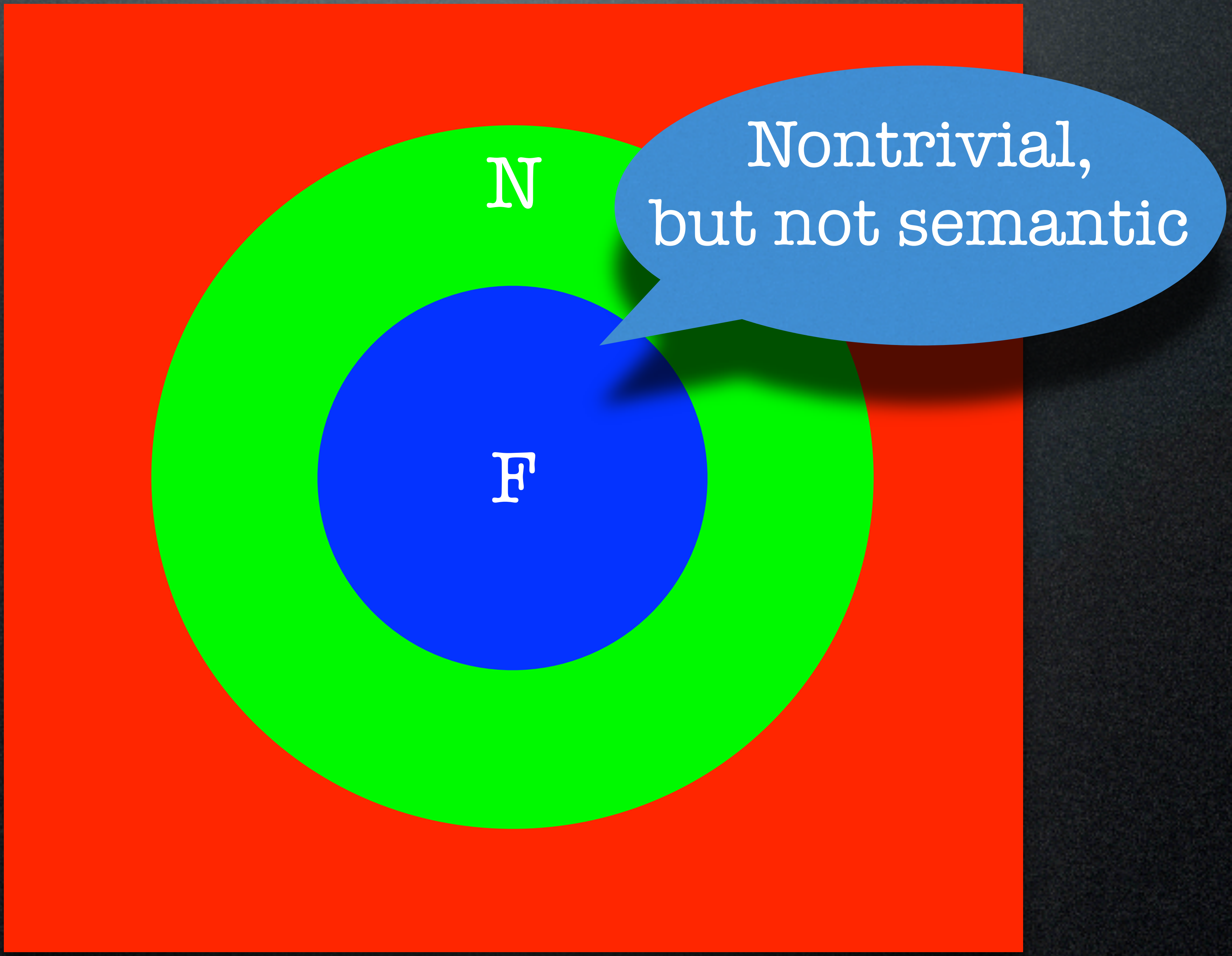


Nontrivial, but not semantic

Static Analysis for N



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Theorem: F is undecidable

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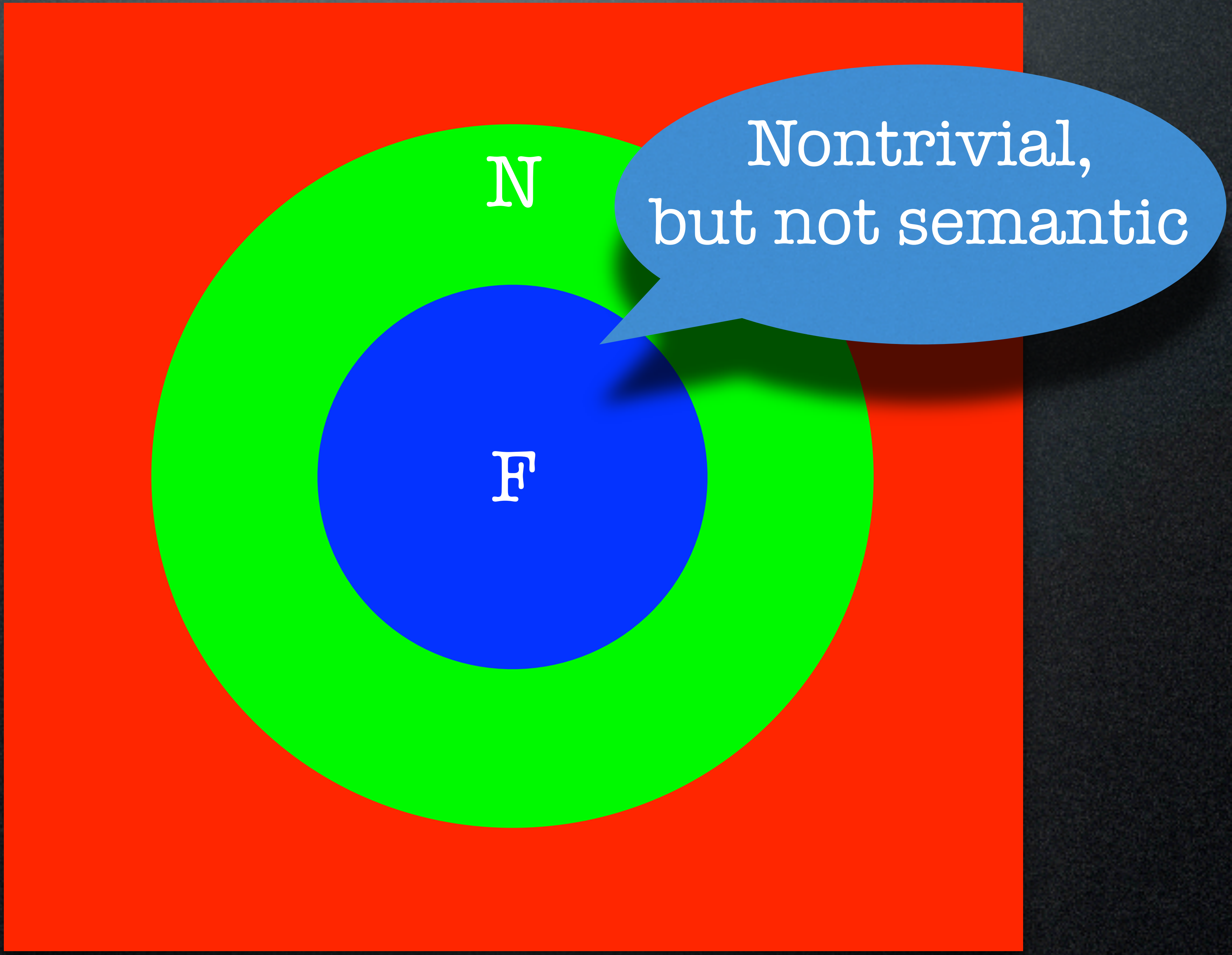

Nontrivial,
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Theorem: F is undecidable

Wells, Typability and Type Checking in the Second-Order λ-Calculus Are Equivalent and Undecidable, LICS 1994

System F for N: Improvability

- Okay for System F to be undecidable, as long as there is no **better** approximation of N that is decidable (**more** efficient).

Recursive inseparability

Definition:
Let $A \subseteq P$. A is **recursively inseparable** from P if there is no B such that $A \subseteq B \subseteq P$ and B is decidable (“recursive”).

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  - There **may** be a (type) system out there that extends System F, guarantees N **and** is decidable.

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  - Does not fo roof

I don't believe it, though

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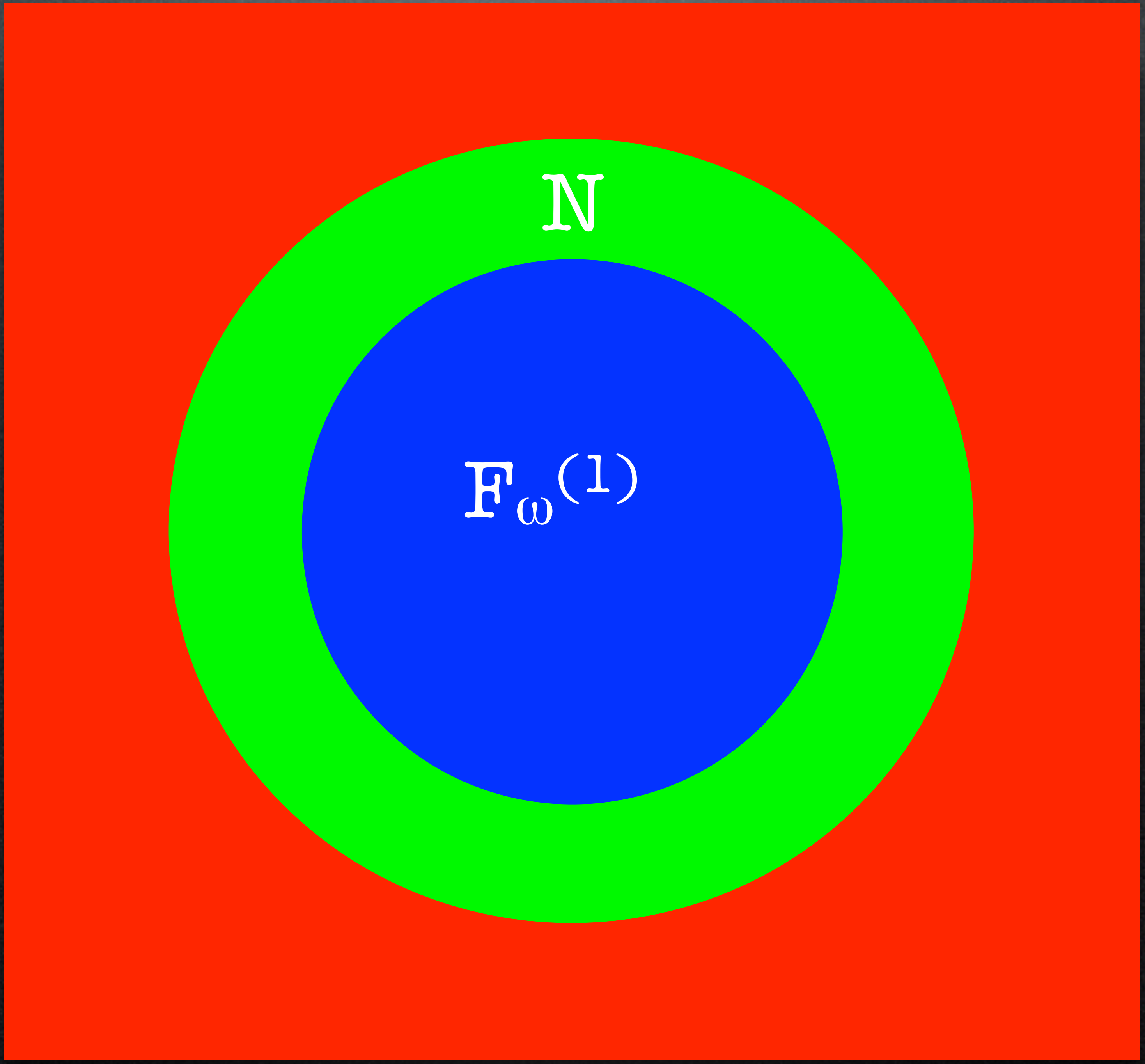
Another analysis for N



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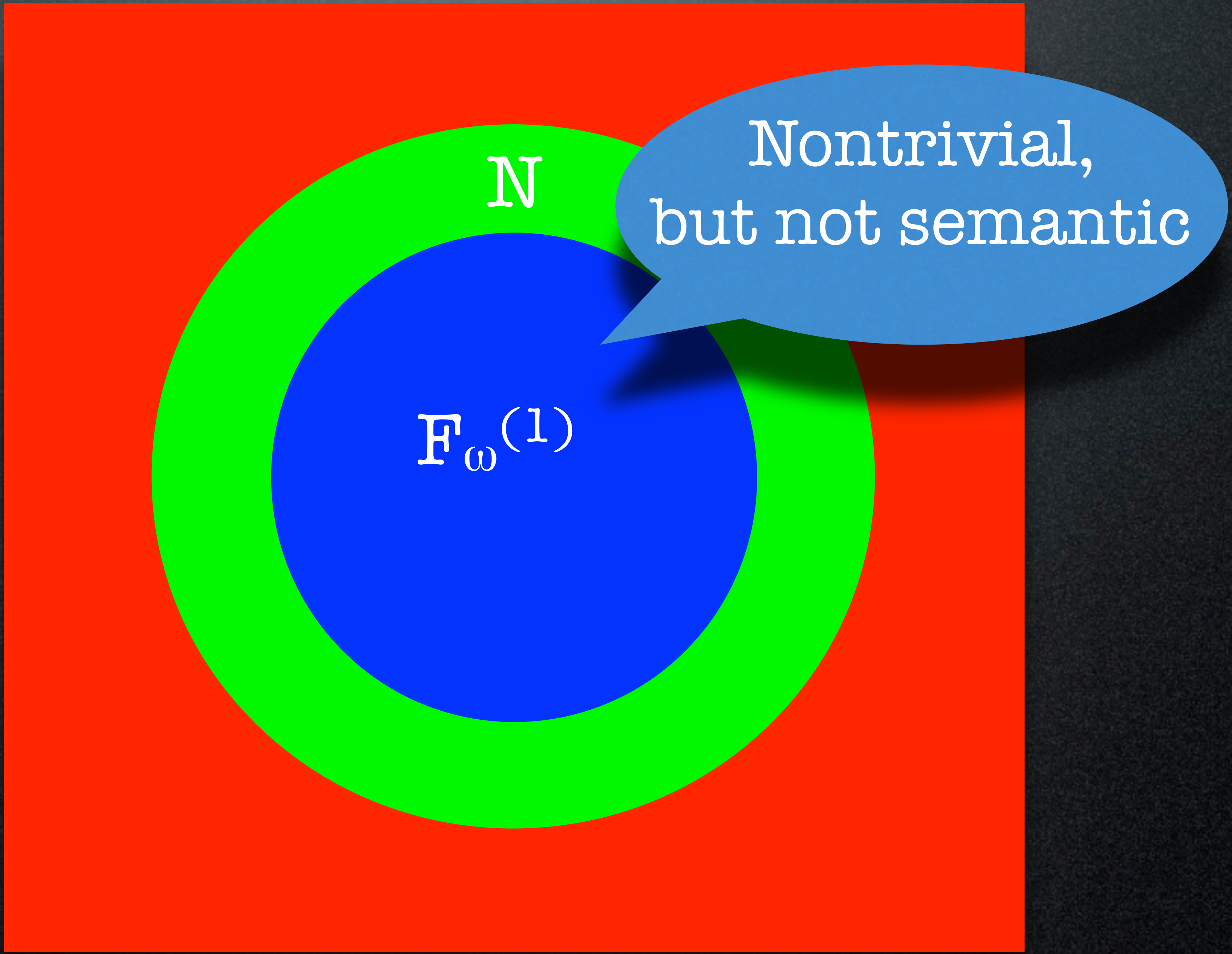

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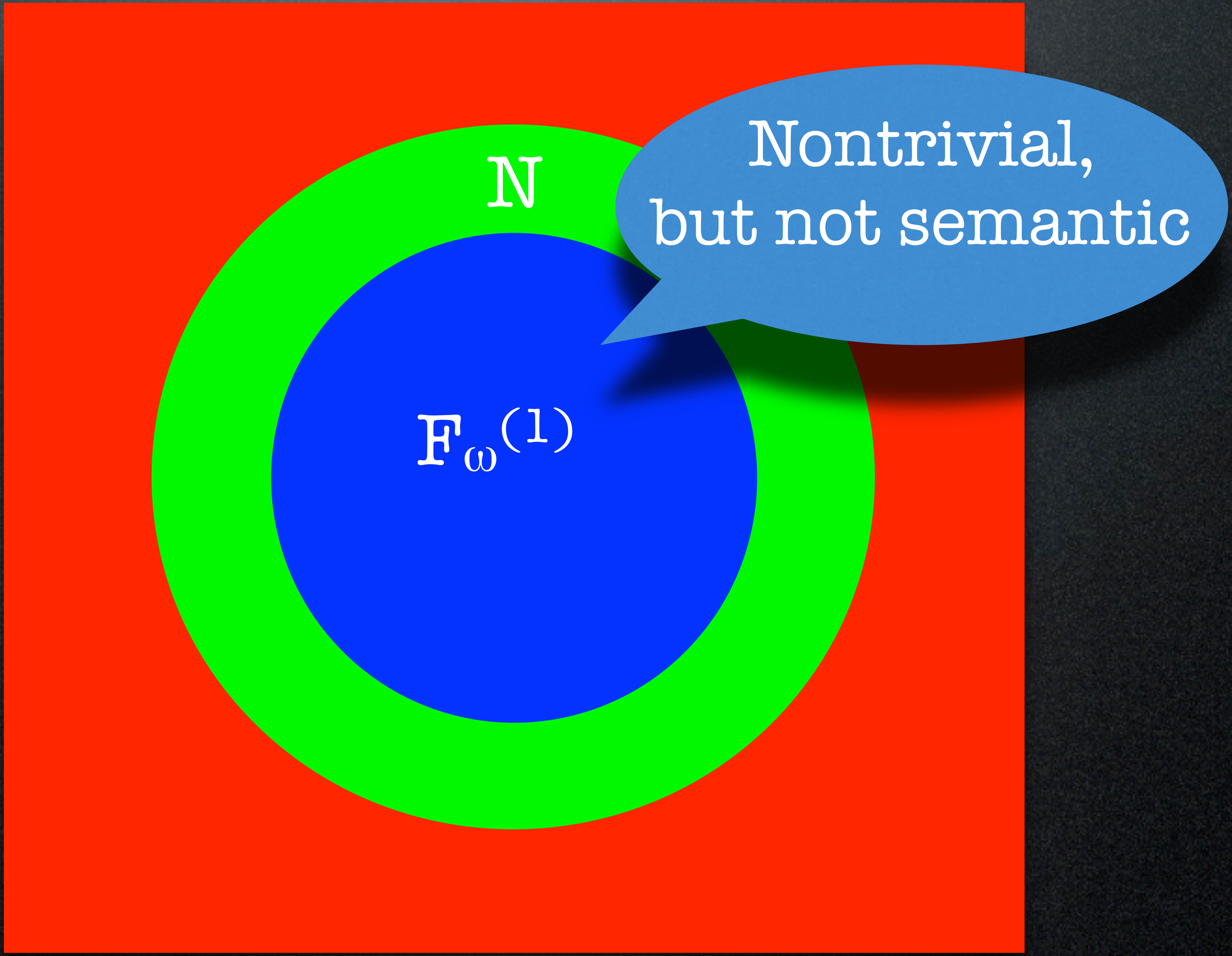

Theorem: $F_\omega^{(1)}$ is undecidable

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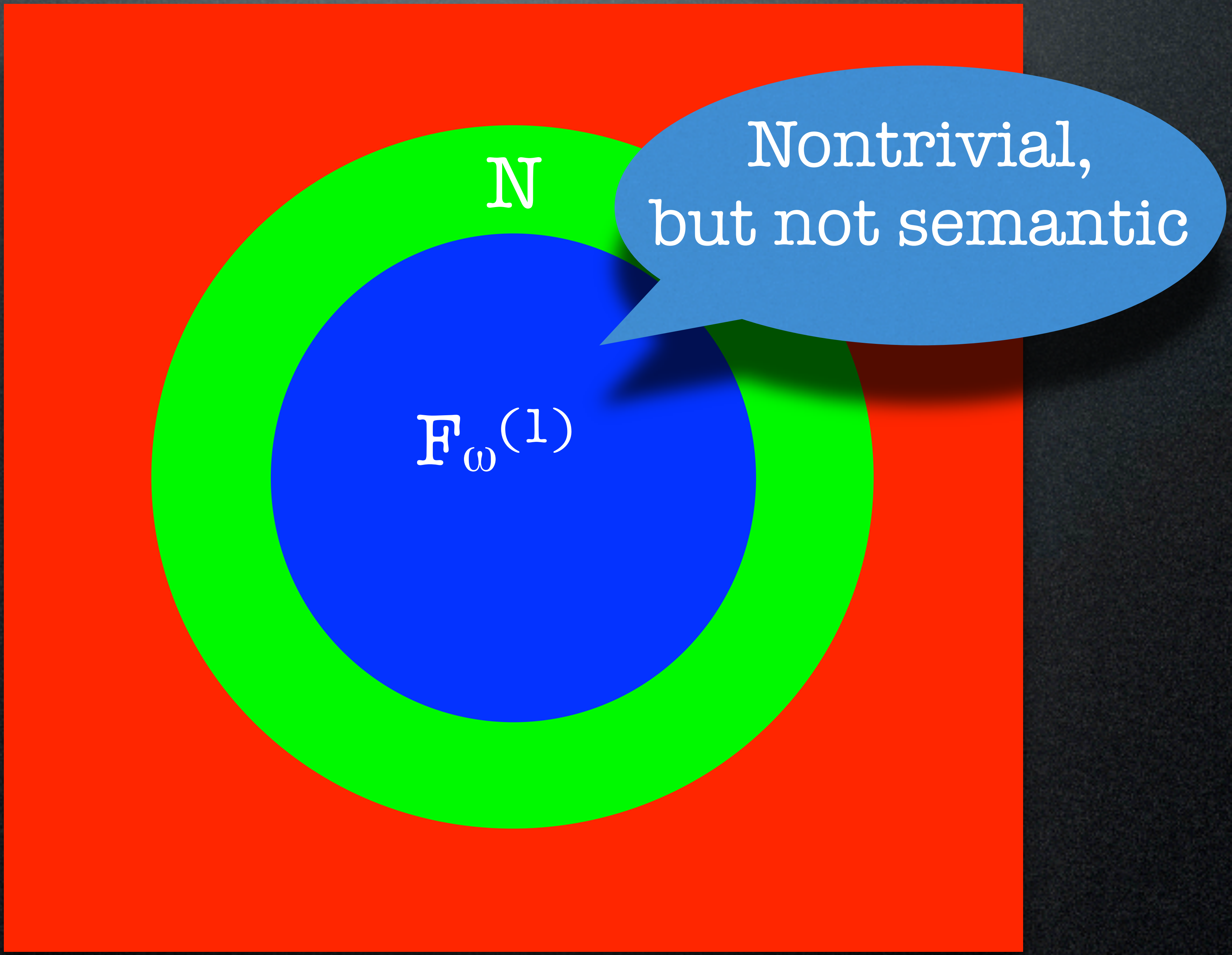

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Urzyczyn, Type reconstruction in Fω, MSCS 1997

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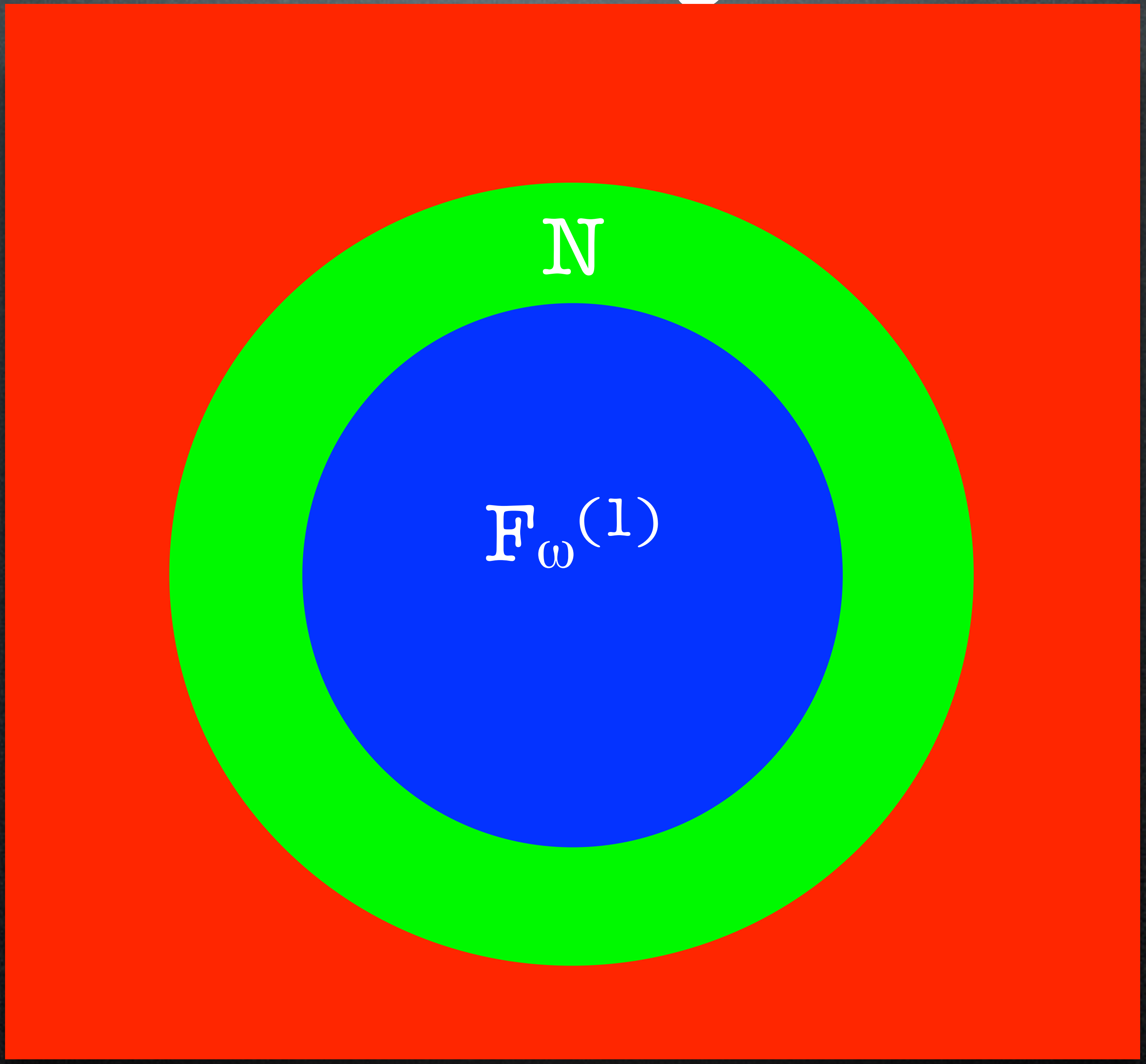

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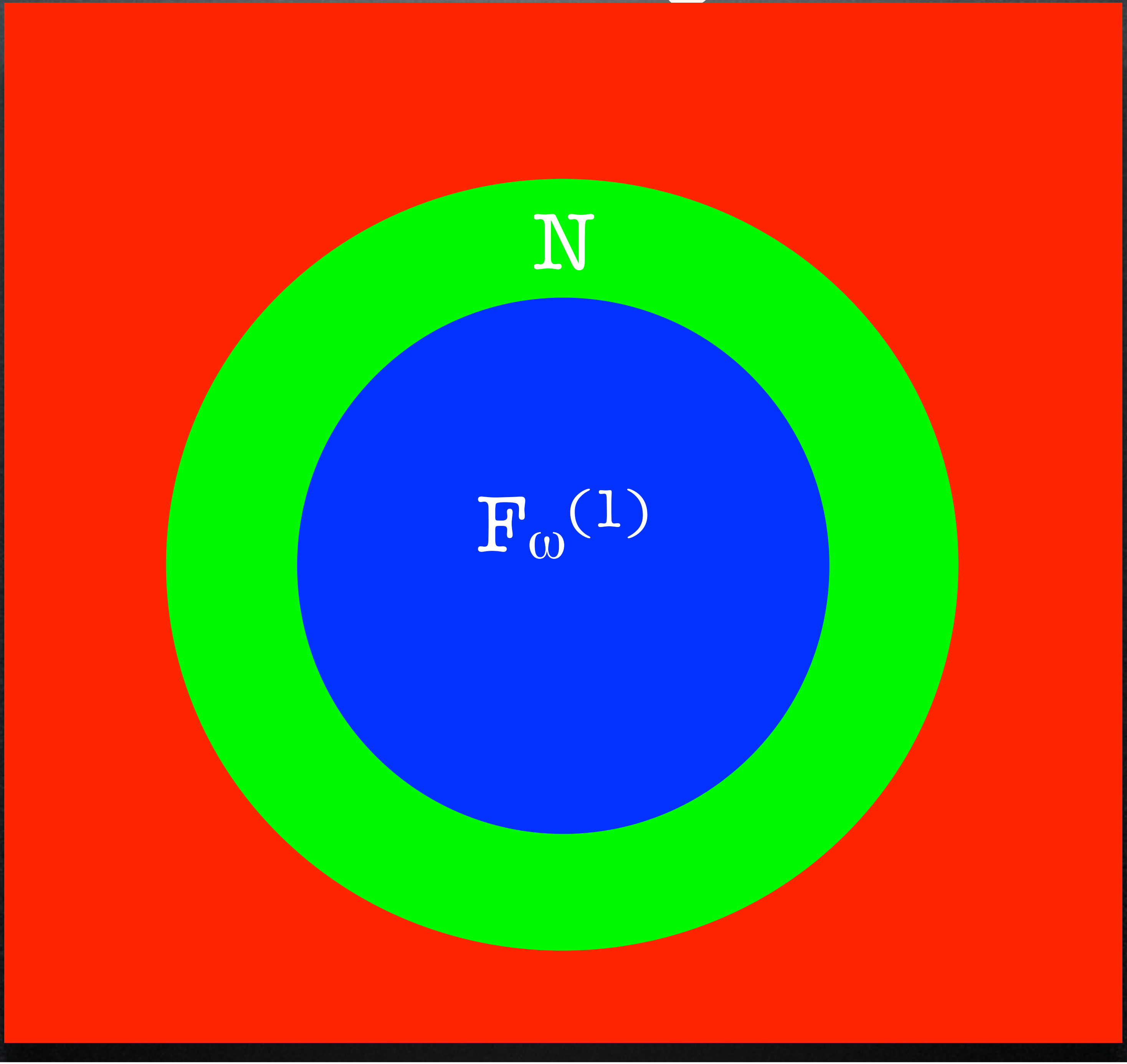

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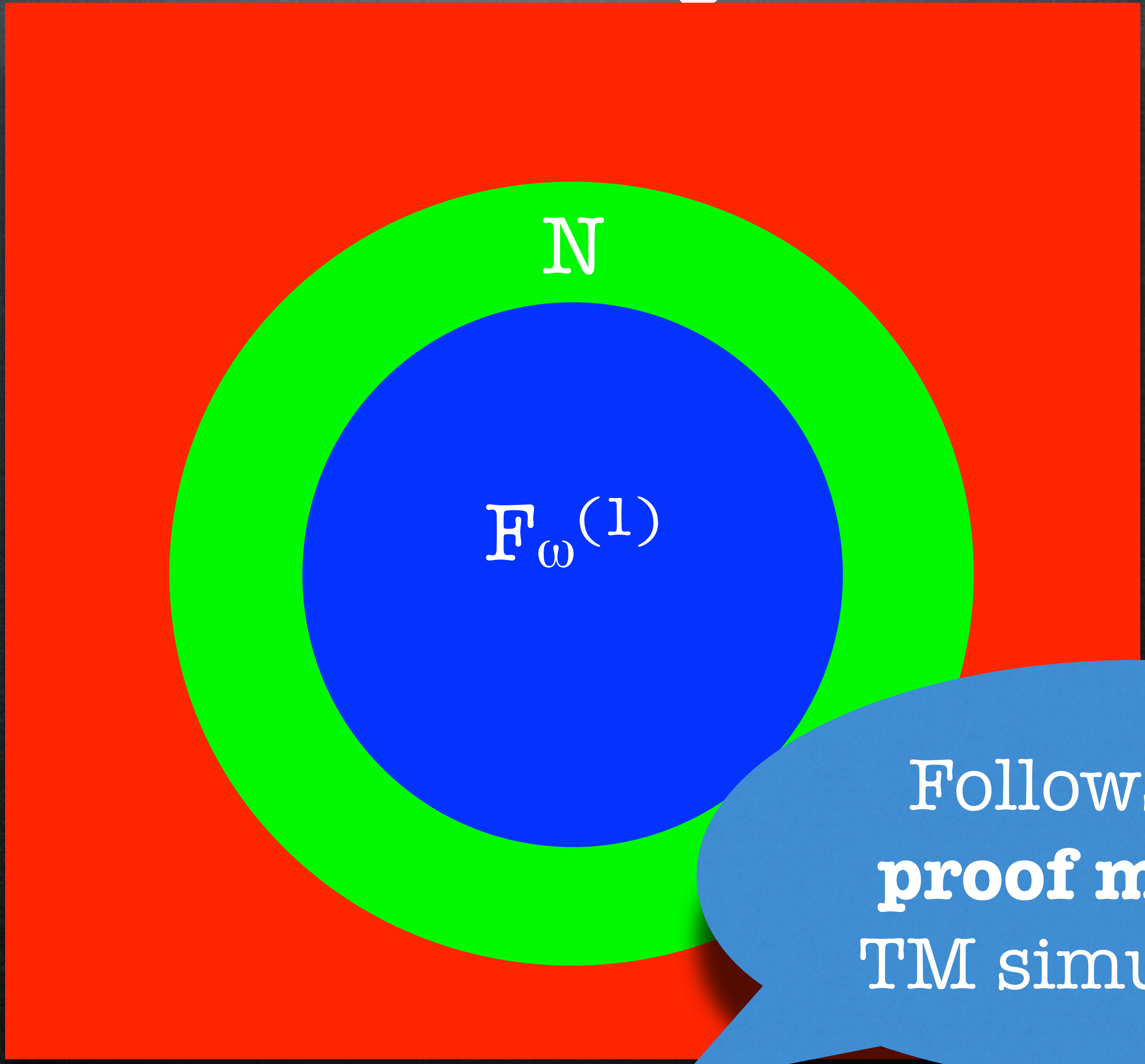
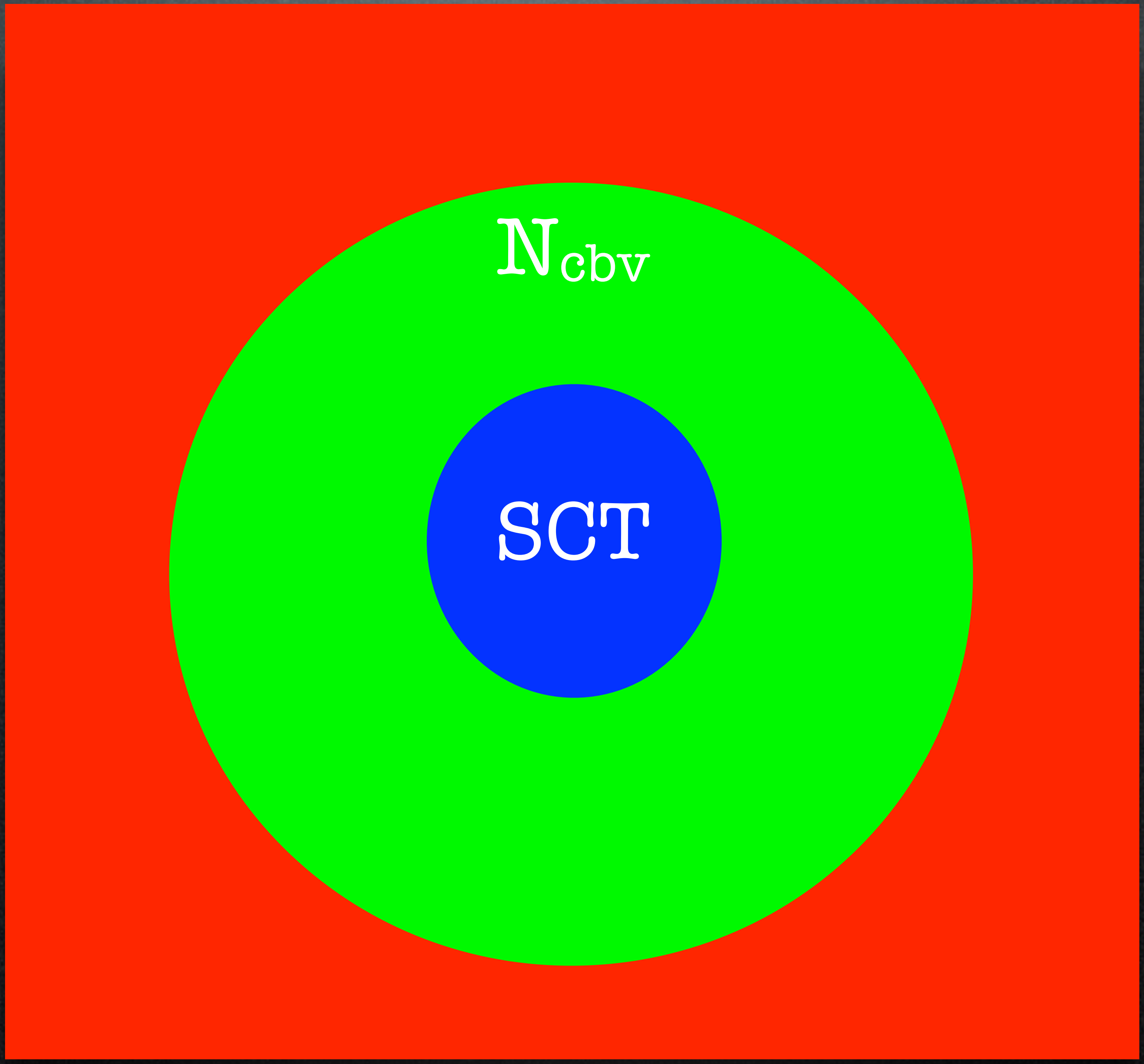


Follows from **proof method**: TM simulation

Theorem: $F_\omega^{(1)}$ is recursively inseparable from N

SCT for N



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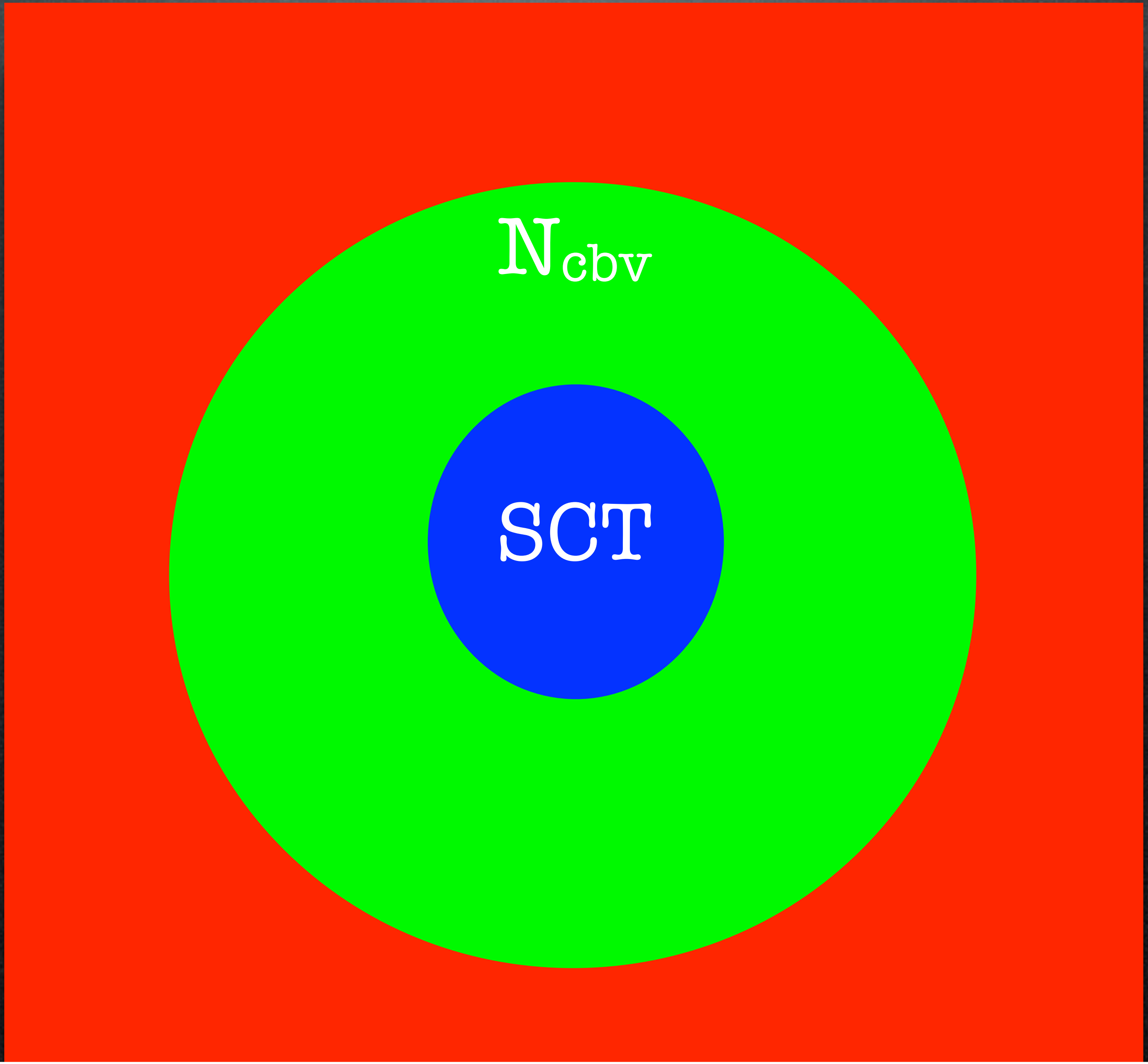

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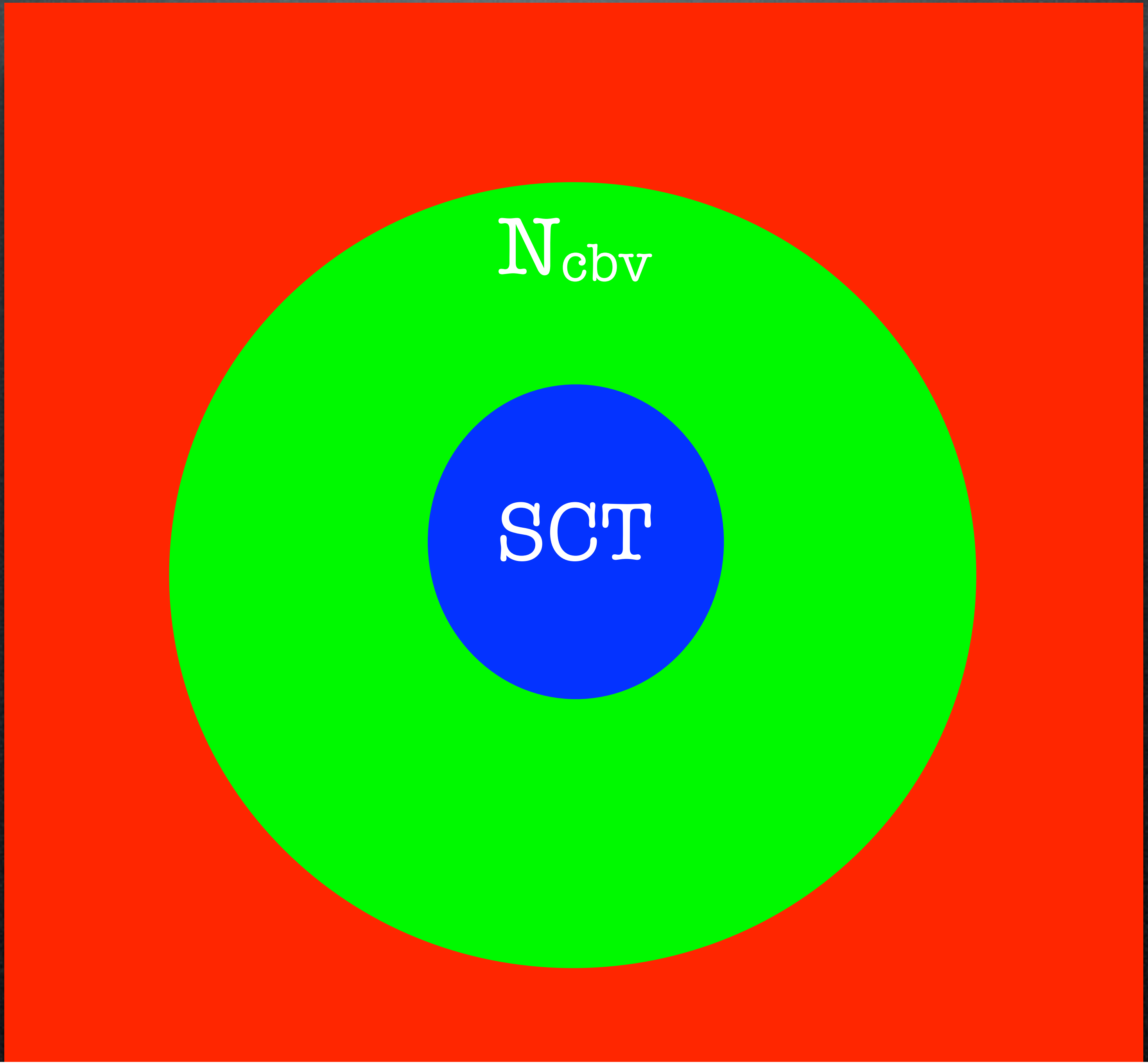
(Complexity?)

SCT for N



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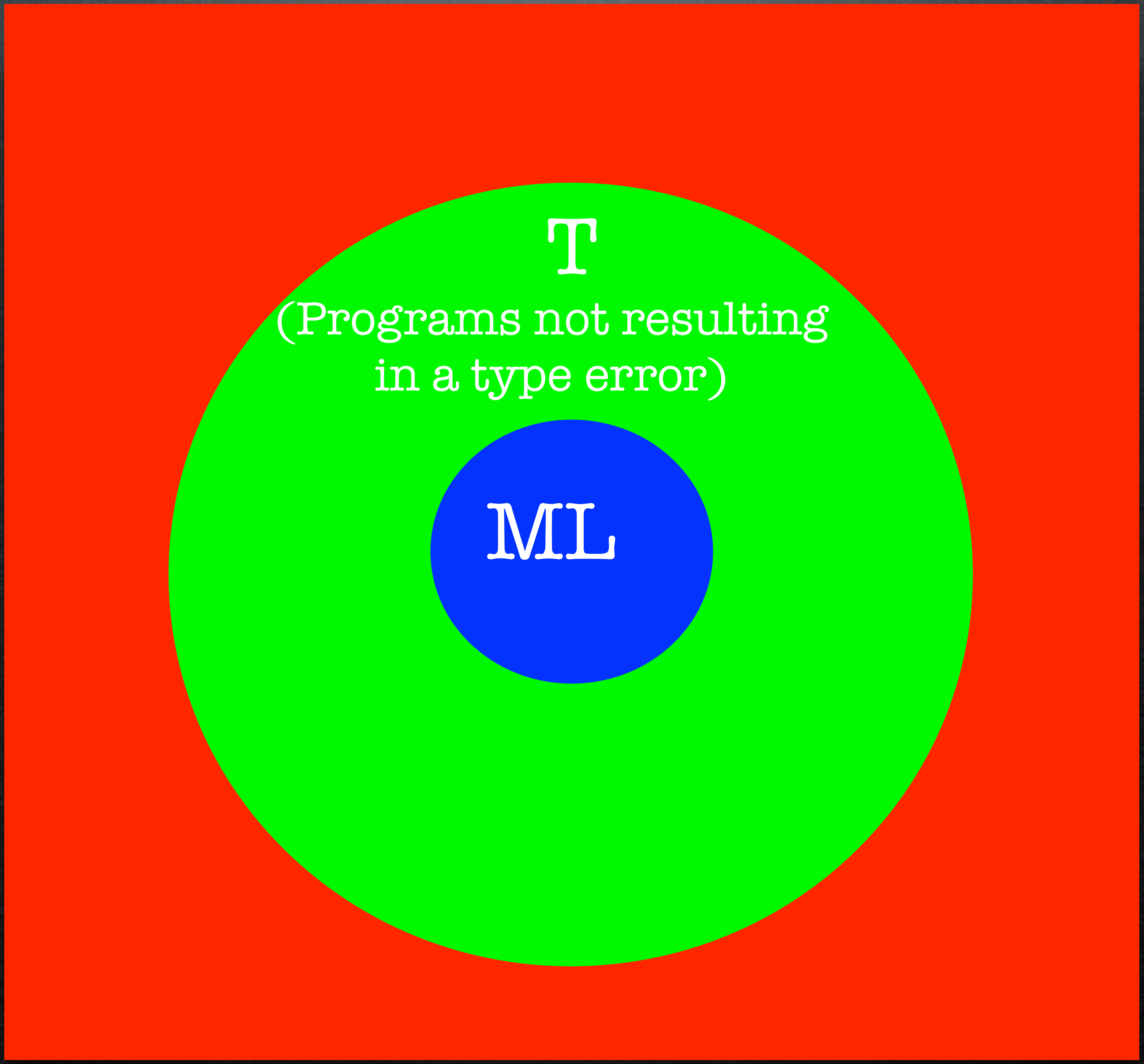
Bohr, Jones, Termination analysis of the untyped lambda-calculus, 2004

An analysis for type error freeness



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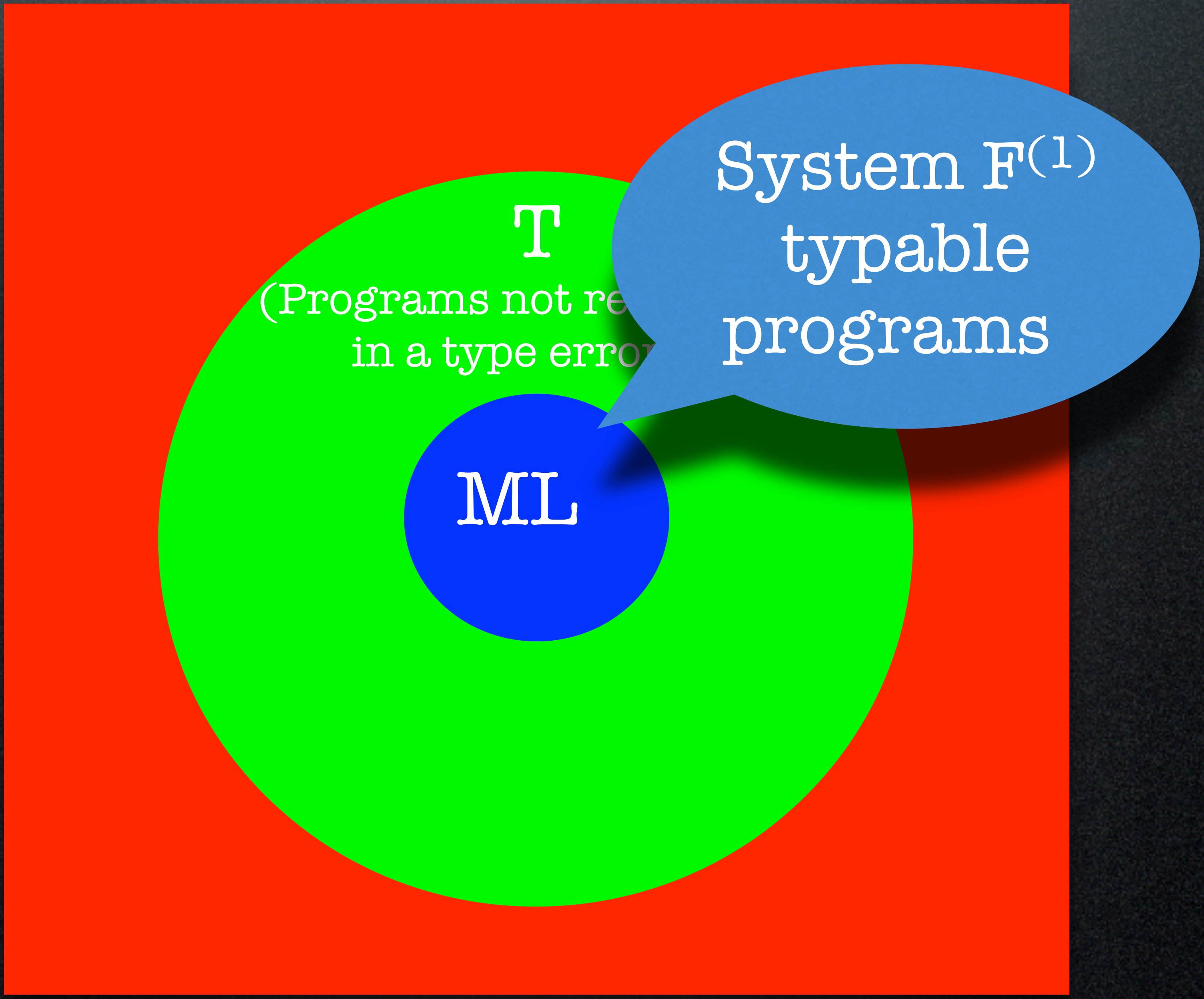

ML goodness

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  - **Invariant** under let-reduction

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ML is “semantic” for let-expressions: Context sensitivity for nonrecursive definitions

ML typability as static analysis for type error freeness

- Is ML typability improvable?

ML typability as static analysis for type error freeness

Theorem: Let ML $\subseteq$ B $\subseteq$ T.
Then B is DEXPTIME-hard.

Henglein, A Lower Bound for Full Polymorphic Type Inference: Girard-Reynolds Typability is DEXPTIME-hard, Utrecht U. TR RUU-CS-90-14, 1990

ML typability as static analysis for type error free

No, ML is not improvable for type error detection

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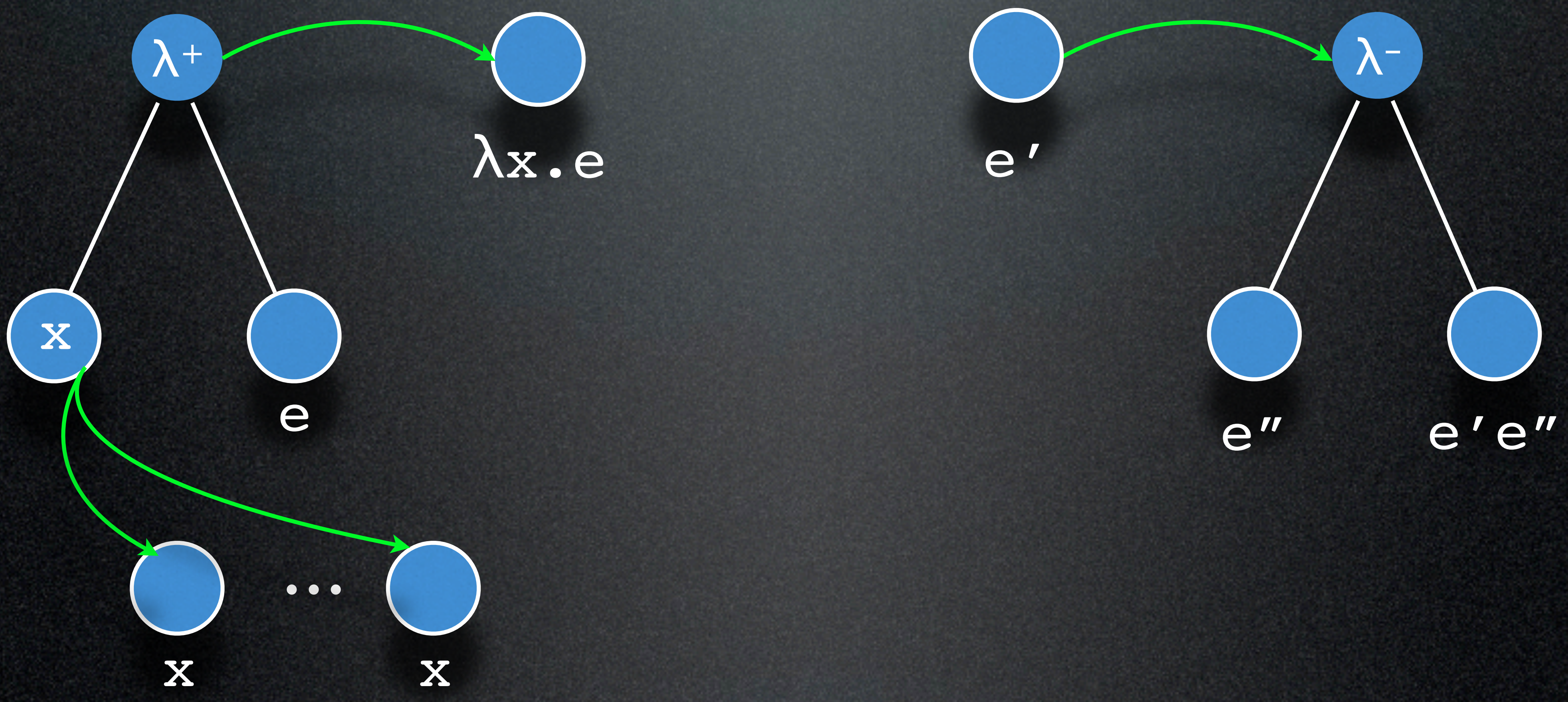
Henglein, A Lower Bound for Full Polymorphic Type Inference: Girard-Reynolds Typability is DEXPTIME-hard, Utrecht U. TR RUU-CS-90-14, 1990

mVFA

0CFA in direct style

Build **graph** with **flow** and **tree** edges. One node per subexpression, plus some extra ones.

1. Base flow rules, resulting in graph G:



mVFA

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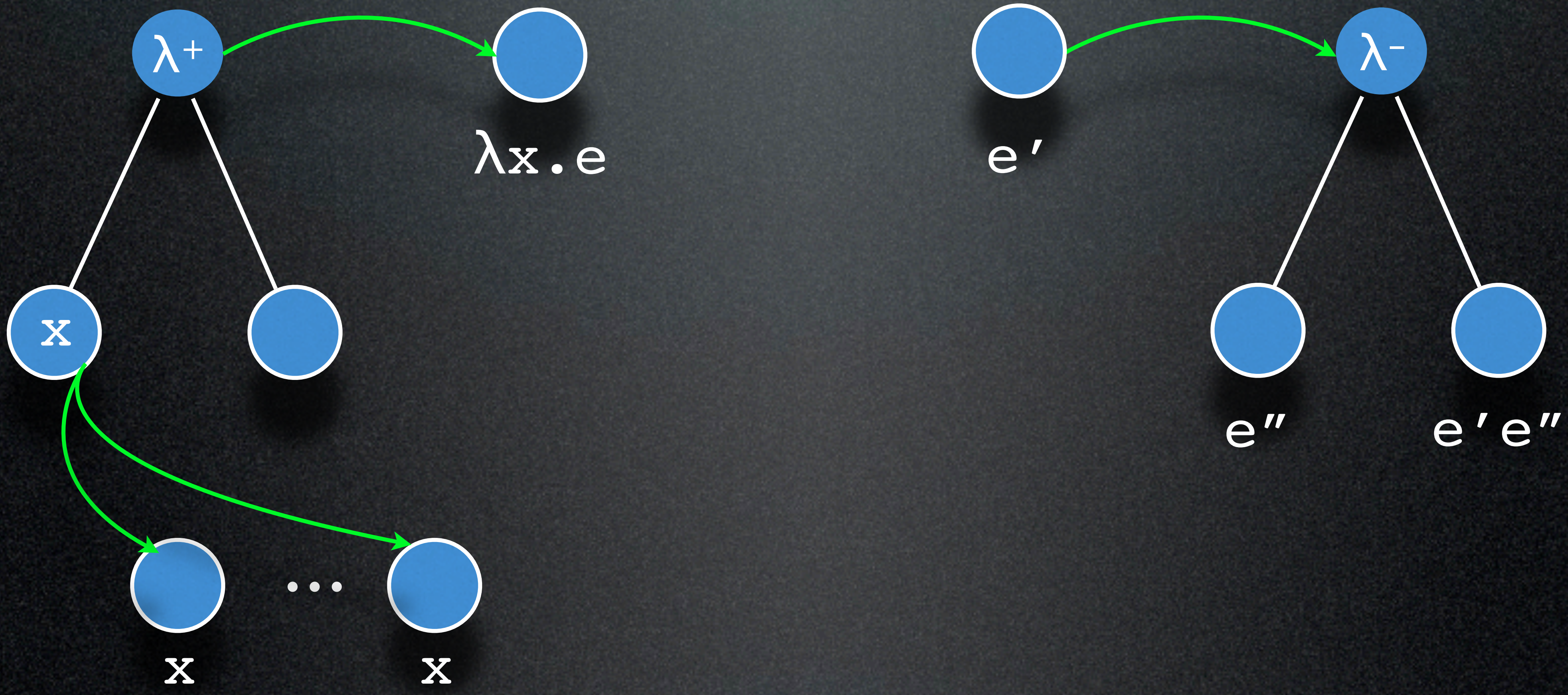
O(n) nodes

O(n) edges

Out- and indegree **1**,λ if affine λ-term



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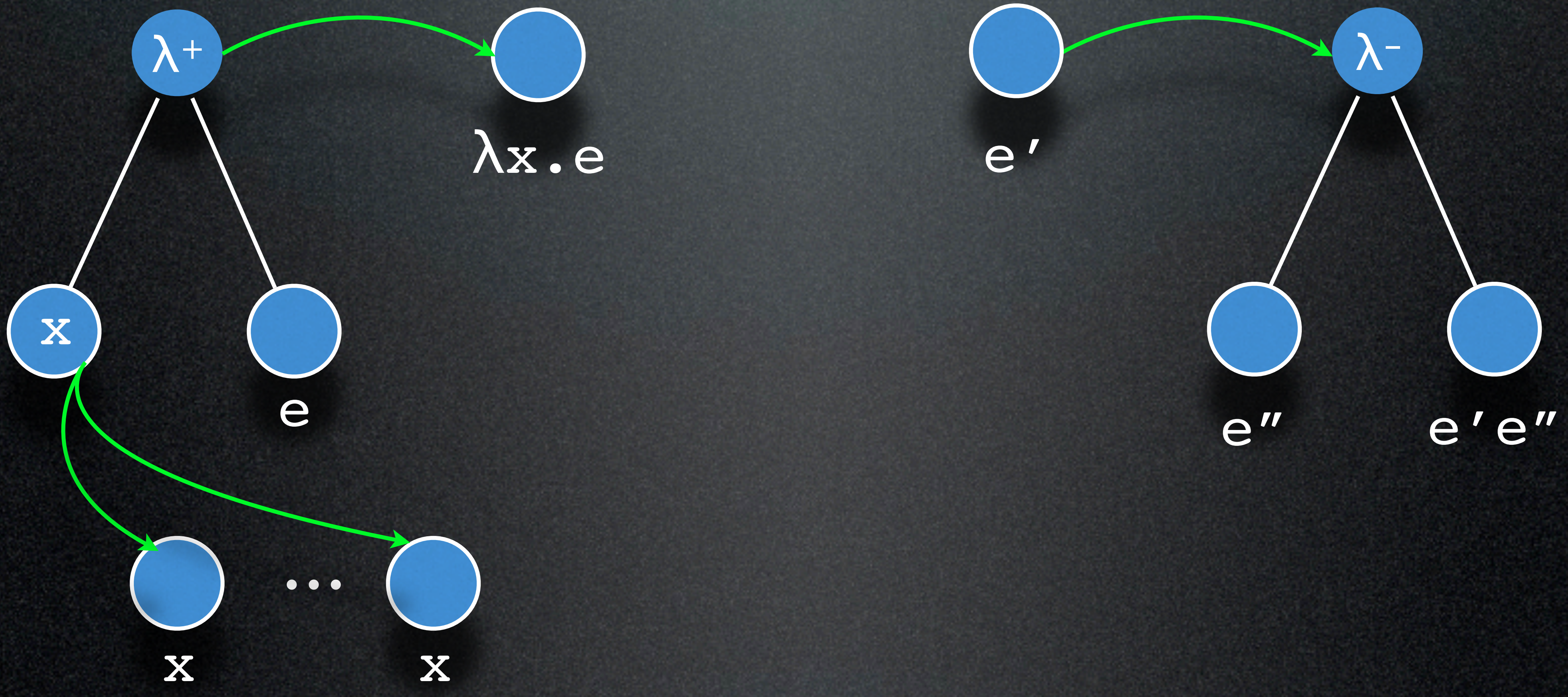
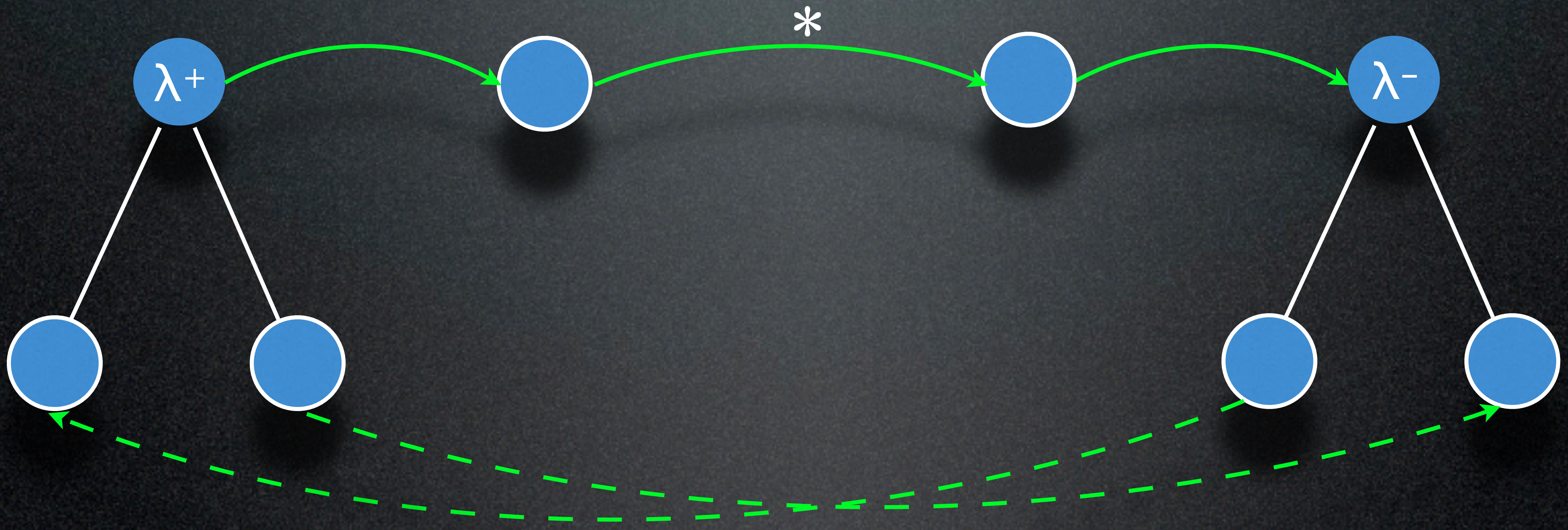


mVFA

0CFA in direct style

2. Closure rule:



mVFA

OCFA in direct style

Algorithm:
Close base graph under closure rule, resulting in graph G.

mVFA

OCFA in direct style

Theorem: mVFA can be implemented in time $O(d\, m^* + p\, n + q)$, where

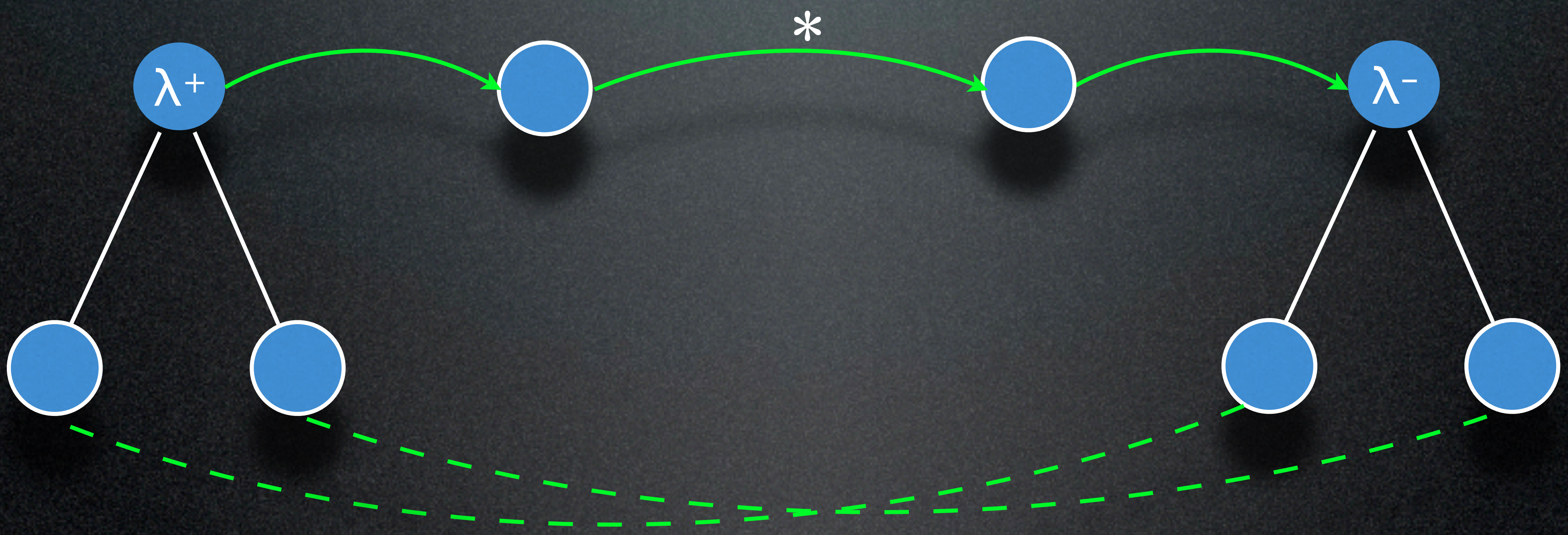
- n: number of nodes
- d: maximum outdegree of G,
- m^*: number of flow edges in G^* (flow-transitive closure of G),
- p: number of closure rule applications.
- q: number of reachability queries

Yellin, Speeding Up Dynamic Transitive Closure for Bounded Degree Graphs,
Acta Informatica 30, 369-384, 1993

sVFA

Simple monomorphic VFA

1. Base rules: As for mVFA
2. Closure rule:



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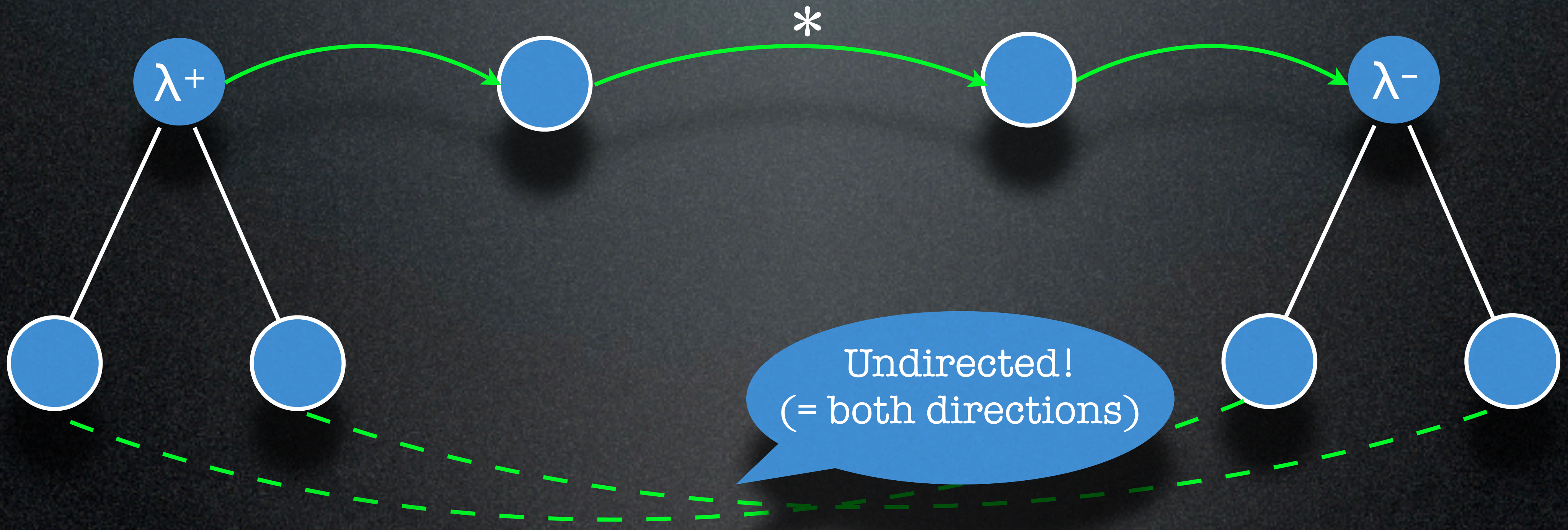

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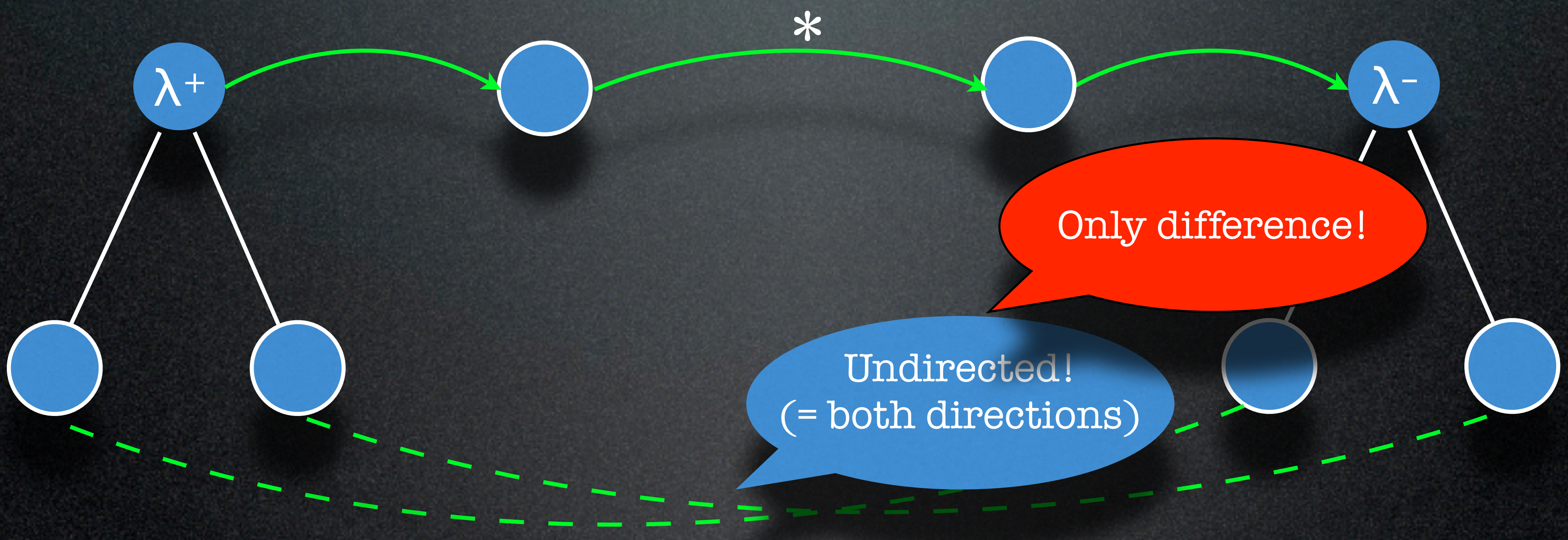

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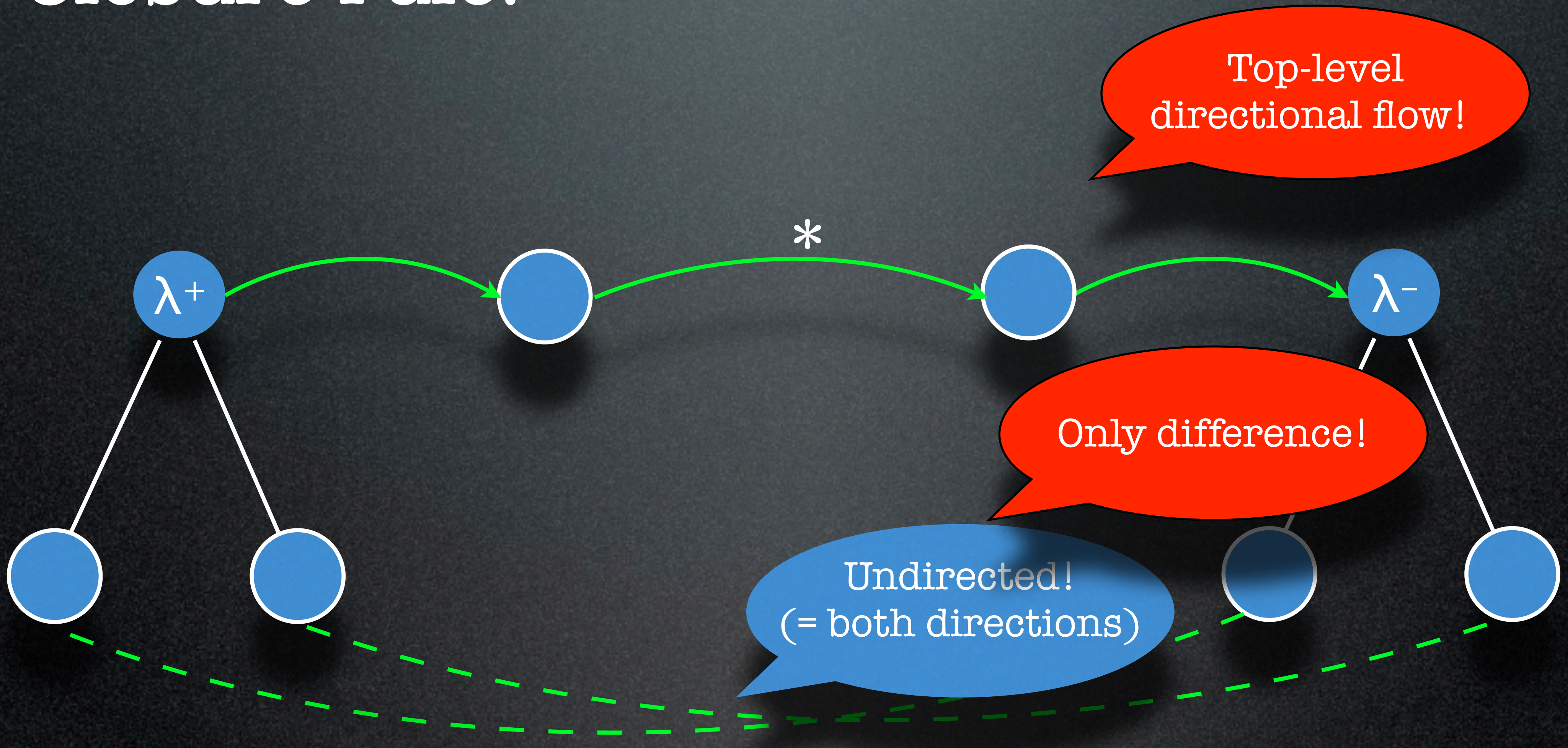

sVFA

Simple monomorphic VFA

Algorithm:
Close base graph under closure rule by unification closure, using union/find data structure.

sVFA

Simple monomorphic VFA

Theorem: sVFA can be implemented in time $O(n\,\alpha(n,n) + q\,n)$, where

- $\alpha(m,n)$: inverse Ackerman function
- q: number of reach set queries

Henglein, Simple Closure Analysis, TOPPS TR D-193, 1992

sVFA

Simple monomorphic VFA

- Very fast in practice
- Applications:
  - Binding-time analysis
    Henglein, Efficient Type Inference for Higher-Order Binding-Time Analysis, FPCA 1991
  - Dynamic type inference for Scheme
    Henglein, Global tagging optimization by type inference, LFP 1992
  - Closure analysis in Similix
    Bondorf, Jørgensen, Efficient Analysis for Realistic Off-Line Partial Evaluation, JFP 1993
- No significant reduction in precision vis a vis mVFA observed

sub0-CFA

…

(similar characterization)

sVFA predictability

sVFA predictability

- sVFA is invariant under

sVFA predictability

- sVFA is invariant under
  - linear beta-reduction

sVFA predictability

- sVFA is invariant under
  - linear beta-reduction
  - eta-reduction (for pure λ-terms)

sVFA predictability

Theorem:
sVFA reachability is P-complete

Van Horn, Mairson, Flow Analysis, Linearity, and PTIME, SAS 2008

sVFA predictability

also for sub0-CFA

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Theorem:
Let B be such that sVFA $\subseteq$ B $\subseteq$ R,
where R is semantic (un)reachability.
Then B is P-hard.

sVFA predictability

also for sub0-CFA

Theorem:
sVFA rea te

Van Horn, Mairson

Follows from proof method used: invariance under linear λ-term reduction

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Adaptiveness

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- Assume S0 $\subseteq$ S1 $\subseteq$ P, with algorithms A0, A1 for S0, S1, respectively.
- A1 is **adaptive** over A0 if its (time) complexity is < 2 times the complexity of A0 on instances from S0.
  - A1 is allowed to take substantially more time than A0 on instances outside S0.

Adaptiveness

- **Intuition**: A static analysis algorithm should **not be slower** on instances where **a less precise** analysis algorithm manages to compute the semantically **correct** result (on “easy instances”).

Questions

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- Are the various kCFA-algorithms adaptive (over sVFA or sub0-CFA)?

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- Is (functional) kCFA improvable for $k \geq 1$?

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