A Definitions

Definition A.1 (environmental bisimulation). *Environmental relation $\mathcal{X}$ is an environmental bisimulation if $P\mathcal{X}_{\mathcal{E}}Q$ implies:*

1. $P \xrightarrow{\tau} P'$ *implies* $Q \xrightarrow{\tau} \dots \xrightarrow{\tau} Q'$ *and* $P'\mathcal{X}_{\mathcal{E}}Q'$
2. $P \xrightarrow{a(V)} P'$ *with* $a\hat{\mathcal{E}}b$ *and* $V\hat{\mathcal{E}}^*W$ *implies* $Q \xrightarrow{\tau} \dots \xrightarrow{b(W)} \dots \xrightarrow{\tau} Q'$ *and* $P'\mathcal{X}_{\mathcal{E}}Q'$
3. $P \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P'$ *with* $a\hat{\mathcal{E}}b$ *and* $\tilde{c} \notin \mathsf{fn}(\#_1(\mathcal{E}))$ *implies* $\exists\tilde{d} \notin \mathsf{fn}(\#_2(\mathcal{E}))$. $Q \xrightarrow{\tau} \dots \xrightarrow{\nu\tilde{d}.\overline{b}\langle W\rangle} \dots \xrightarrow{\tau} Q'$ *and* $P'\mathcal{X}_{\mathcal{E}\cup\{(V,W)\}}Q'$
4. *the converse of (1-3) on* Q
5. $V_1\hat{\mathcal{E}}W_1$ *and* $V_2\hat{\mathcal{E}}W_2$ *imply* $V_1 = V_2 \iff W_1 = W_2$
6. $`(P')\hat{\mathcal{E}}`(Q')$ *implies* $P|P'\mathcal{X}_{\mathcal{E}}Q|Q'$
7. $P\mathcal{X}_{\mathcal{E}\cup\{(a,b)\}}Q$ *for any* $a \notin \mathsf{fn}(P, \#_1(\mathcal{E}))$ *and* $b \notin \mathsf{fn}(Q, \#_2(\mathcal{E}))$
8. $V\hat{\mathcal{E}}W$ *implies:*
   (a) $V = a$ *implies* $W = b$
   (b) $V = f$ *implies* $W = f$
   (c) $V = \hat{f}(V_1, \dots, V_l)$ *implies* $W = \hat{g}(W_1, \dots, W_m)$
   (d) $V \in \mathbf{Quo}$ *implies* $\exists b \notin \mathsf{fn}(\mathcal{E}, P, Q)$. $P\mathcal{X}_{\mathcal{E}\cup\{(\mathit{reify}_b(V),\mathit{reify}_b(W))\}}Q$
9. *the converse of 8 on* W

Definition A.2 (context closure for environmental bisimulations). *We write* $P\mathcal{X}^{(*)}_{\mathcal{E}}Q$ *if* $P \equiv \nu\tilde{c}.(P_0|P_1)$ *and* $Q \equiv \nu\tilde{d}.(Q_0|Q_1)$ *with* $P_0\mathcal{X}_{\mathcal{E}'}Q_0$ *and* $P_1\hat{\mathcal{E}'}^*Q_1$ *for an environmental relation* $\mathcal{X}$*, where* $\tilde{c} \notin \mathsf{fn}(\#_1(\mathcal{E}))$ *and* $\tilde{d} \notin \mathsf{fn}(\#_2(\mathcal{E}))$ *with* $\hat{\mathcal{E}} \subseteq \{(V,W) \mid V\hat{\mathcal{E}'}^*W$ *and* $\mathsf{fn}(V) \cap \{\tilde{c}\} = \mathsf{fn}(W) \cap \{\tilde{d}\} = \emptyset\}$.

Definition A.3 (environmental bisimulation up-to context). *Environmental relation $\mathcal{X}$ is an environmental bisimulation up-to context if $P\mathcal{X}_{\mathcal{E}}Q$ implies:*

1. $P \xrightarrow{\tau} P'$ *implies* $Q \xrightarrow{\tau} \dots \xrightarrow{\tau} Q'$ *and* $P'\mathcal{X}^{(*)}_{\mathcal{E}}Q'$
2. $P \xrightarrow{a(V)} P'$ *with* $a\hat{\mathcal{E}}b$ *and* $V\hat{\mathcal{E}}^*W$ *implies* $Q \xrightarrow{\tau} \dots \xrightarrow{b(W)} \dots \xrightarrow{\tau} Q'$ *and* $P'\mathcal{X}^{(*)}_{\mathcal{E}}Q'$
3. $P \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P'$ *with* $a\hat{\mathcal{E}}b$ *and* $\tilde{c} \notin \mathsf{fn}(\#_1(\mathcal{E}))$ *implies* $\exists\tilde{d} \notin \mathsf{fn}(\#_2(\mathcal{E}))$. $Q \xrightarrow{\tau} \dots \xrightarrow{\nu\tilde{d}.\overline{b}\langle W\rangle} \dots \xrightarrow{\tau} Q'$ *and* $P'\mathcal{X}^{(*)}_{\mathcal{E}\cup\{(V,W)\}}Q'$
4. *the converse of (1-3) on* Q
5. $V_1\hat{\mathcal{E}}W_1$ *and* $V_2\hat{\mathcal{E}}W_2$ *imply* $V_1 = V_2 \iff W_1 = W_2$

6. *$`(P')\hat{\mathcal{E}}`(Q')$ implies $P|P'\mathcal{X}^{(*)}_{\mathcal{E}}Q|Q'$*

7. *$P\mathcal{X}_{\mathcal{E}\cup\{(a,b)\}}Q$ for any $a \notin \mathsf{fn}(P, \#_1(\mathcal{E}))$ and $b \notin \mathsf{fn}(Q, \#_2(\mathcal{E}))$*

8. *$V\hat{\mathcal{E}}W$ implies:*

   (a) *$V = a$ implies $W = b$*

   (b) *$V = f$ implies $W = f$*

   (c) *$V = \hat{f}(V_1, \ldots, V_l)$ implies $W = \hat{g}(W_1, \ldots, W_m)$*

   (d) *$V \in \mathbf{Quo}$ implies $\exists b \notin \mathsf{fn}(\mathcal{E}, P, Q).\ P\mathcal{X}^{(*)}_{\mathcal{E}\cup\{(\mathit{reify}_b(V), \mathit{reify}_b(W))\}}Q$*

9. *the converse of 8 on W*

Structural equivalence. Define evaluation contexts by $C ::= [] \mid (C|P) \mid (P|C) \mid \nu c.C$. Structural equivalence $\equiv$ is the smallest equivalence relation on processes that is closed under evaluation contexts, with:

$$\begin{array}{cc} P \equiv P|0 & P_1|(P_2|P_3) \equiv (P_1|P_2)|P_3 \\ P_1|P_2 \equiv P_2|P_1 & !P \equiv P|!P \\ \nu a.0 \equiv 0 & \nu a.\nu b.P \equiv \nu b.\nu a.P \\ \multicolumn{2}{c}{P_1|\nu a.P_2 \equiv \nu a.(P_1|P_2) \text{ if } a \notin \mathsf{fn}(P_1)} \end{array}$$

B Proofs

B.1 Reduction respects structural equivalence

Lemma B.1 (reduction respects structural equivalence).

1. *$P \equiv Q$ and $P \xrightarrow{\alpha} P'$ imply $Q \xrightarrow{\alpha} Q'$ and $P' \equiv Q'$*

2. *$P \equiv Q$ and $Q \xrightarrow{\alpha} Q'$ imply $P \xrightarrow{\alpha} P'$ and $P' \equiv Q'$.*

Proof. By induction on the derivation of structural equivalence. Since the proof of clause 2 is similar to that of clause 1, we omit the proof of clause 2. We have some cases of transition derivation.

Case: $P \equiv P|0 \quad P \xrightarrow{\alpha} P'$
We have $P|0 \xrightarrow{\alpha} P'|0$ by PAR-L. Then we have $P' \equiv P'|0$

Case: $P_1|(P_2|P_3) \equiv (P_1|P_2)|P_3 \quad P_1|(P_2|P_3) \xrightarrow{\alpha} P'$
There are 9 subcases of the transition derivation of $P_1|(P_2|P_3) \xrightarrow{\alpha} P'$.
Subcase PAR-L: $P_1 \xrightarrow{\alpha} P_1'$
We have $P_1|(P_2|P_3) \xrightarrow{\alpha} P_1'|(P_2|P_3)$ where $\mathsf{bn}(\alpha) \cap \mathsf{fn}(P_2|P_3) = \emptyset$. Then by PAR-L, $P_1|P_2 \xrightarrow{\alpha} P_1'|P_2$, since $\mathsf{bn}(\alpha) \cup \mathsf{fn}(P_2) = \emptyset$. Again by PAR-L, $(P_1|P_2)|P_3 \xrightarrow{\alpha} (P_1'|P_2)|P_3$, since $\mathsf{bn}(\alpha) \cup \mathsf{fn}(P_3) = \emptyset$. By definition of structural equivalence, we have $P_1'|(P_2|P_3) \equiv (P_1'|P_2)|P_3$.
Subcase PAR-R, PAR-L: $P_2 \xrightarrow{\alpha} P_2'$
We have $P_1|(P_2|P_3) \xrightarrow{\alpha} P_1|(P_2'|P_3)$ where $\mathsf{bn}(\alpha) \cap \mathsf{fn}(P_1, P_3) = \emptyset$. Then by PAR-R, $P_1|P_2 \xrightarrow{\alpha} P_1|P_2'$,

since $\mathsf{bn}(\alpha) \cup \mathsf{fn}(P_1) = \emptyset$. Again by PAR-L, $(P_1|P_2)|P_3 \xrightarrow{\alpha} (P_1|P_2')|P_3$, since $\mathsf{bn}(\alpha) \cup \mathsf{fn}(P_3) = \emptyset$. By definition of structural equivalence, we have $P_1|(P_2'|P_3) \equiv (P_1|P_2')|P_3$.

Subcase PAR-R, PAR-R: $P_3 \xrightarrow{\alpha} P_3'$

We have $P_1|(P_2|P_3) \xrightarrow{\alpha} P_1|(P_2|P_3')$ where $\mathsf{bn}(\alpha) \cap \mathsf{fn}(P_1, P_2) = \emptyset$. Then by PAR-R, $(P_1|P_2)|P_3 \xrightarrow{\alpha} (P_1|P_2)|P_3'$, since $\mathsf{bn}(\alpha) \cup \mathsf{fn}(P_1|P_2) = \emptyset$. By definition of structural equivalence, we have $P_1|(P_2|P_3') \equiv (P_1|P_2)|P_3'$.

Subcase PAR-R, TAU-L: $P_2 \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P_2' \quad P_3 \xrightarrow{a(V)} P_3'$

We have $P_1|(P_2|P_3) \xrightarrow{\tau} P_1|\nu\tilde{c}.(P_2'|P_3')$ where $\{\tilde{c}\} \cap \mathsf{fn}(P_3) = \emptyset$. Then by PAR-R, for some $\tilde{c_1}$ s.t. $\{\tilde{c_1}\} \cap \mathsf{fn}(P_1, P_2, P_3) = \emptyset$, $P_1|P_2 \xrightarrow{\nu\tilde{c}.\overline{a}\langle\{\tilde{c_1}/\tilde{c}\}V\rangle} P_1|\{\tilde{c_1}/\tilde{c}\}P_2'$. By TAU-L, we have $(P_1|P_2)|P_3 \xrightarrow{\tau} \nu\tilde{c_1}.((P_1|\{\tilde{c_1}/\tilde{c}\}P_2')|\{\tilde{c_1}/\tilde{c}\}P_3')$. By $\{\tilde{c_1}\} \cap \mathsf{fn}(P_1) = \emptyset$ and definition of structural equivalence, we have $P_1|\nu\tilde{c}.(P_2'|P_3') \equiv \nu\tilde{c_1}.(P_1|(\{\tilde{c_1}/\tilde{c}\}P_2'|\{\tilde{c_1}/\tilde{c}\}P_3')) \equiv \nu\tilde{c_1}.((P_1|\{\tilde{c_1}/\tilde{c}\}P_2')|\{\tilde{c_1}/\tilde{c}\}P_3')$. Therefore $P_1|\nu\tilde{c}.(P_2'|P_3') \equiv \nu\tilde{c_1}.((P_1|\{\tilde{c_1}/\tilde{c}\}P_2')|\{\tilde{c_1}/\tilde{c}\}P_3')$.

Subcase PAR-R, TAU-R: $P_2 \xrightarrow{a(V)} P_2' \quad P_3 \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P_3'$

We have $P_1|(P_2|P_3) \xrightarrow{\tau} P_1|\nu\tilde{c}.(P_2'|P_3')$ where $\{\tilde{c}\} \cap \mathsf{fn}(P_2) = \emptyset$. Then by PAR-R, for some $\tilde{c_1}$ s.t. $\{\tilde{c_1}\} \cap \mathsf{fn}(P_1, P_2, P_3) = \emptyset$, $P_1|P_2 \xrightarrow{a(\{\tilde{c_1}/\tilde{c}\}V)} P_1|\{\tilde{c_1}/\tilde{c}\}P_2'$. By TAU-R, we have $(P_1|P_2)|P_3 \xrightarrow{\tau} \nu\tilde{c_1}.((P_1|\{\tilde{c_1}/\tilde{c}\}P_2')|\{\tilde{c_1}/\tilde{c}\}P_3')$. By $\{\tilde{c_1}\} \cap \mathsf{fn}(P_1) = \emptyset$ and definition of structural equivalence, we have $P_1|\nu\tilde{c}.(P_2'|P_3') \equiv \nu\tilde{c_1}.(P_1|(\{\tilde{c_1}/\tilde{c}\}P_2'|\{\tilde{c_1}/\tilde{c}\}P_3')) \equiv \nu\tilde{c_1}.((P_1|\{\tilde{c_1}/\tilde{c}\}P_2')|\{\tilde{c_1}/\tilde{c}\}P_3')$. Therefore $P_1|\nu\tilde{c}.(P_2'|P_3') \equiv \nu\tilde{c_1}.((P_1|\{\tilde{c_1}/\tilde{c}\}P_2')|\{\tilde{c_1}/\tilde{c}\}P_3')$.

Subcase TAU-L, PAR-L: $P_1 \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P_1' \quad P_2 \xrightarrow{a(V)} P_2'$

We have $P_1|(P_2|P_3) \xrightarrow{\tau} \nu\tilde{c}.(P_1'|(P_2'|P_3))$ where $\{\tilde{c}\} \cap \mathsf{fn}(P_2, P_3) = \emptyset$. Then by TAU-L, $P_1|P_2 \xrightarrow{\tau} \nu\tilde{c}.(P_1'|P_2')$. By PAR-L, we have $(P_1|P_2)|P_3 \xrightarrow{\tau} \nu\tilde{c}.(P_1'|P_2')|P_3$. By $\{\tilde{c}\} \cap \mathsf{fn}(P_3) = \emptyset$ and definition of structural equivalence, we have $\nu\tilde{c}.(P_1'|(P_2'|P_3)) \equiv \nu\tilde{c}.((P_1'|P_2')|P_3) \equiv \nu\tilde{c}.(P_1'|P_2')|P_3$. Therefore $\nu\tilde{c}.(P_1'|(P_2'|P_3)) \equiv \nu\tilde{c}.(P_1'|P_2')|P_3$.

Subcase TAU-L, PAR-R: $P_1 \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P_1' \quad P_3 \xrightarrow{a(V)} P_3'$

We have $P_1|(P_2|P_3) \xrightarrow{\tau} \nu\tilde{c}.(P_1'|(P_2|P_3'))$ where $\{\tilde{c}\} \cap \mathsf{fn}(P_2, P_3) = \emptyset$. Then by PAR-L, $P_1|P_2 \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P_1'|P_2$. By TAU-L, we have $(P_1|P_2)|P_3 \xrightarrow{\tau} \nu\tilde{c}.((P_1'|P_2)|P_3')$. By definition of structural equivalence, we have $\nu\tilde{c}.(P_1'|(P_2|P_3')) \equiv \nu\tilde{c_1}.((P_1'|P_2)|P_3')$.

Subcase TAU-R, PAR-L: $P_1 \xrightarrow{a(V)} P_1' \quad P_2 \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P_2'$

We have $P_1|(P_2|P_3) \xrightarrow{\tau} \nu\tilde{c}.(P_1'|(P_2'|P_3))$ where $\{\tilde{c}\} \cap \mathsf{fn}(P_1, P_3) = \emptyset$. Then by TAU-R, $P_1|P_2 \xrightarrow{\tau} \nu\tilde{c}.(P_1'|P_2')$. By PAR-L, we have $(P_1|P_2)|P_3 \xrightarrow{\tau} \nu\tilde{c}.(P_1'|P_2')|P_3$. By $\{\tilde{c}\} \cap \mathsf{fn}(P_3) = \emptyset$ and definition of structural equivalence, we have $\nu\tilde{c}.(P_1'|(P_2'|P_3)) \equiv \nu\tilde{c}.((P_1'|P_2')|P_3) \equiv \nu\tilde{c}.(P_1'|P_2')|P_3$. Therefore $\nu\tilde{c}.(P_1'|(P_2'|P_3)) \equiv \nu\tilde{c}.(P_1'|P_2')|P_3$.

Subcase TAU-R, PAR-R: $P_1 \xrightarrow{a(V)} P_1' \quad P_3 \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P_3'$

We have $P_1|(P_2|P_3) \xrightarrow{\tau} \nu\tilde{c}.(P_1'|(P_2|P_3'))$ where $\{\tilde{c}\} \cap \mathsf{fn}(P_1, P_2) = \emptyset$. Then by PAR-L, $P_1|P_2 \xrightarrow{a(V)} P_1'|P_2$. By TAU-L, we have $(P_1|P_2)|P_3 \xrightarrow{\tau} \nu\tilde{c}.((P_1'|P_2)|P_3')$. By definition of structural equivalence, we have $\nu\tilde{c}.(P_1'|(P_2|P_3')) \equiv \nu\tilde{c_1}.((P_1'|P_2)|P_3')$.

Case: $P_1|P_2 \equiv P_2|P_1$

Subcase PAR-L: $P_1 \xrightarrow{\alpha} P_1'$

We have $P_1|P_2 \xrightarrow{\alpha} P_1'|P_2$, where $\mathsf{bn}(\alpha) \cap \mathsf{fn}(P_2) = \emptyset$. Then by PAR-R, we have $P_2|P_1 \xrightarrow{\alpha} P_2|P_1'$. By definition of structural equivalence, we have $P_1'|P_2 \equiv P_2|P_1'$.

Subcase PAR-R: $P_2 \xrightarrow{\alpha} P_2'$
Similar.

Subcase TAU-L: $P_1 \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P_1' \quad P_2 \xrightarrow{a(V)} P_2'$
We have $P_1|P_2 \xrightarrow{\tau} \nu\tilde{c}.(P_1'|P_2')$, where $\{\tilde{c}\} \cap \mathsf{fn}(P_2) = \emptyset$. Then by TAU-R, we have $P_2|P_1 \xrightarrow{\tau} \nu\tilde{c}.(P_2'|P_1')$. By definition of structural equivalence, we have $\nu\tilde{c}.(P_1'|P_2') \equiv \nu\tilde{c}.(P_2'|P_1')$.

Subcase TAU-R: $P_1 \xrightarrow{a(V)} P_1' \quad P_2 \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P_2'$
Similar.

Case: $!P \equiv P|!P$
Assume $!P \xrightarrow{\alpha} P'$. By REP, we have $P|!P \xrightarrow{\alpha} P'$. Of course $P' \equiv P'$ and that's all.

Case: $\nu a.0 \equiv 0$
The transition of $\nu a.0$ can't happen.

Case: $\nu a.\nu b.P \equiv \nu b.\nu a.P$
It is immediate by the case $a = b$, so we suppose $a \neq b$. There are 4 subcases of transition derivations of $\nu a.\nu b.P$.

Subcase SCOPE, SCOPE: $\nu b.P \xrightarrow{\alpha} \nu b.P'$
Assume $\nu a.\nu b.P \xrightarrow{\alpha} \nu a.\nu b.P'$ We have $P \xrightarrow{\alpha} P'$, where $a, b \notin \mathsf{n}(\alpha)$. Then we have $\nu b.\nu a.P \xrightarrow{\alpha} \nu b.\nu a.P'$ By definition of structural equivalence, we have $\nu a.\nu b.P' \equiv \nu b.\nu a.P'$.

Subcase SCOPE, OPEN: $\nu b.P \xrightarrow{\nu\tilde{c},b.\overline{a_1}\langle V\rangle} P'$
Assume $\nu a.\nu b.P \xrightarrow{\nu\tilde{c},b.\overline{a_1}\langle V\rangle} \nu a.P'$, where $a \notin \mathsf{n}(\nu\tilde{c},b.\overline{a_1}\langle V\rangle)$. We have $P \xrightarrow{\nu\tilde{c}.\overline{a_1}\langle V\rangle} P'$, where $b \neq a_1, b \in \mathsf{fn}(V) \setminus \{\tilde{c}\}$. Then we have $\nu a.P \xrightarrow{\nu\tilde{c}.\overline{a_1}\langle V\rangle} \nu a.P'$ by SCOPE, and thus $\nu b.\nu a.P \xrightarrow{\nu\tilde{c},b.\overline{a_1}\langle V\rangle} \nu a.P'$ by OPEN. Naturally we have $\nu a.P' \equiv \nu a.P'$.

Subcase OPEN, SCOPE: $\nu b.P \xrightarrow{\nu\tilde{c}.\overline{a_1}\langle V\rangle} \nu b.P'$
Assume $\nu a.\nu b.P \xrightarrow{\nu\tilde{c},a.\overline{a_1}\langle V\rangle} \nu b.P'$, where $a \neq a_1, a \in \mathsf{fn}(V) \setminus \{\tilde{c}\}$. We have $P \xrightarrow{\nu\tilde{c}.\overline{a_1}\langle V\rangle} P'$, where $b \notin \mathsf{n}(\nu\tilde{c}.\overline{a_1}\langle V\rangle)$. Then we have $\nu a.P \xrightarrow{\nu\tilde{c},a.\overline{a_1}\langle V\rangle} P'$ by OPEN, and thus $\nu b.\nu a.P \xrightarrow{\nu\tilde{c},a.\overline{a_1}\langle V\rangle} \nu b.P'$ by SCOPE. Naturally we have $\nu b.P' \equiv \nu b.P'$.

Subcase OPEN, OPEN: $\nu b.P \xrightarrow{\nu\tilde{c},b.\overline{a_1}\langle V\rangle} P'$
Assume $\nu a.\nu b.P \xrightarrow{\nu\tilde{c},a,b.\overline{a_1}\langle V\rangle} P'$, where $a, b \neq a_1, a, b \in \mathsf{fn}(V) \setminus \{\tilde{c}\}$. We have $P \xrightarrow{\nu\tilde{c}.\overline{a_1}\langle V\rangle} P'$. Then we have $\nu a.P \xrightarrow{\nu\tilde{c},a.\overline{a_1}\langle V\rangle} P'$ by OPEN, and thus $\nu b.\nu a.P \xrightarrow{\nu\tilde{c},a,b.\overline{a_1}\langle V\rangle} P'$ by OPEN. Naturally we have $P' \equiv P'$.

Case: $P_1|\nu a.P_2 \equiv \nu a.(P_1|P_2)$
There are 6 subcases of transition derivations of $P_1|\nu a.P_2$.

Subcase PAR-L: $P_1 \xrightarrow{\alpha} P_1'$
We have $P_1|\nu a.P_2 \xrightarrow{\alpha} P_1'|\nu a.P_2$, where $\mathsf{bn}(\alpha) \cap \mathsf{fn}(\nu a.P_2) = \emptyset$. Then for some a_1, we have $P_1|\{^{a_1}/_a\}P_2 \xrightarrow{\alpha} P_1'|\{^{a_1}/_a\}P_2$, where $a_1 \notin (\mathsf{bn}(\alpha) \cup \mathsf{fn}(P_1))$. Therefore $\nu a.(P_1|P_2) = \nu a_1.(P_1|\{^{a_1}/_a\}P_2) \xrightarrow{\alpha} \nu a_1.(P_1'|\{^{a_1}/_a\}P_2)$. Then we have $P_1'|\nu a.P_2 \equiv \nu a.(P_1'|P_2) = \nu a_1.(P_1'|\{^{a_1}/_a\}P_2)$. Hence $P_1'|\nu a.P_2 \equiv \nu a_1.(P_1'|\{^{a_1}/_a\}P_2)$.

Subcase PAR-R, SCOPE: $\nu a.P_2 \xrightarrow{\alpha} \nu a.P_2'$
We have $P_1|\nu a.P_2 \xrightarrow{\alpha} P_1|\nu a.P_2'$, where $\mathsf{bn}(\alpha) \cap \mathsf{fn}(P_1) = \emptyset$. We also have $P_2 \xrightarrow{\alpha} P_2'$, where $a \notin \mathsf{n}(\alpha)$.

Then we have $P_1|P_2 \xrightarrow{\alpha} P_1|P_2'$ and thus $\nu a.(P_1|P_2) \xrightarrow{\alpha} \nu a.(P_1|P_2')$ By $a \notin \mathsf{fn}(P_1)$, we have $P_1|\nu a.P_2' \equiv \nu a.(P_1|P_2')$.

Subcase PAR-R, OPEN: $\nu a.P_2 \xrightarrow{\nu\tilde{c},a.\overline{a_1}\langle V\rangle} P_2'$

We have $P_1|\nu a.P_2 \xrightarrow{\nu\tilde{c},a.\overline{a_1}\langle V\rangle} P_1|P_2'$, where $\tilde{c}, a \notin \mathsf{fn}(P_1)$. We also have $P_2 \xrightarrow{\nu\tilde{c}.\overline{a_1}\langle V\rangle} P_2'$, where $a \neq a_1, a \in \mathsf{fn}(V)$. Then we have $P_1|P_2 \xrightarrow{\nu\tilde{c}.\overline{a_1}\langle V\rangle} P_1|P_2'$ and thus $\nu a.(P_1|P_2) \xrightarrow{\nu\tilde{c},a.\overline{a_1}\langle V\rangle} P_1|P_2'$. Naturally we have $P_1|P_2' \equiv P_1|P_2'$.

Subcase TAU-L: $P_1 \xrightarrow{\nu\tilde{c}.\overline{a_1}\langle V\rangle} P_1' \quad \nu a.P_2 \xrightarrow{a_1(V)} \nu a.P_2'$

We have $P_1|\nu a.P_2 \xrightarrow{\tau} \nu\tilde{c}.(P_1'|\nu a.P_2')$, where $\{\tilde{c}\} \cap \mathsf{fn}(\nu a.P_2) = \emptyset$. We also have $P_2 \xrightarrow{a_1(V)} P_2'$, where $a \notin \mathsf{n}(a_1(V))$. Then for some a_2, we have $a_2 \notin \mathsf{fn}(P_1)$, $P_1|\{^{a_2}/_a\}P_2 \xrightarrow{\tau} \nu\tilde{c}.(P_1'|\{^{a_2}/_a\}P_2')$ and thus $\nu a_2.(P_1|\{^{a_2}/_a\}P_2) \xrightarrow{\tau} \nu a_2, \tilde{c}.(P_1'|\{^{a_2}/_a\}P_2')$. By $a_2 \notin \mathsf{fn}(P_1')$, we have $\nu\tilde{c}.(P_1'|\nu a.P_2') \equiv \nu\tilde{c}.(P_1'|\nu a_2.\{^{a_2}/_a\}P_2') \equiv \nu\tilde{c}, a_2.(P_1'|\{^{a_2}/_a\}P_2')$. Therefore $\nu\tilde{c}.(P_1'|\nu a.P_2') \equiv \nu\tilde{c}, a_2.(P_1'|\{^{a_2}/_a\}P_2')$.

Subcase TAU-R, SCOPE: $P_1 \xrightarrow{a(V)} P_1' \quad \nu a.P_2 \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} \nu a.P_2'$

We have $P_1|\nu a.P_2 \xrightarrow{\tau} \nu\tilde{c}.(P_1'|\nu a.P_2')$, where $\{\tilde{c}\} \cap \mathsf{fn}(P_1) = \emptyset$. We also have $P_2 \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P_2'$, where $a \notin \mathsf{n}(\nu\tilde{c}.\overline{a}\langle V\rangle)$. Then we have $P_1|P_2 \xrightarrow{\tau} \nu\tilde{c}.(P_1'|P_2')$ and thus $\nu a.(P_1|P_2) \xrightarrow{\tau} \nu a, \tilde{c}.(P_1'|P_2')$. By $a \notin \mathsf{fn}(P_1)$, we have $\nu\tilde{c}.(P_1'|\nu a.P_2') \equiv \nu\tilde{c}, a.(P_1'|P_2') \equiv \nu a, \tilde{c}.(P_1'|P_2')$. Therefore $\nu\tilde{c}.(P_1'|\nu a.P_2') \equiv \nu a, \tilde{c}.(P_1'|P_2')$.

Subcase TAU-R, OPEN: $P_1 \xrightarrow{a(V)} P_1' \quad \nu a.P_2 \xrightarrow{\nu\tilde{c},a.\overline{a_1}\langle V\rangle} P_2'$

We have $P_1|\nu a.P_2 \xrightarrow{\tau} \nu\tilde{c}, a.(P_1'|P_2')$, where $\tilde{c}, a \notin \mathsf{fn}(P_1)$. We also have $P_2 \xrightarrow{\nu\tilde{c}.\overline{a_1}\langle V\rangle} P_2'$, where $a \neq a_1, a \in \mathsf{fn}(V) \setminus \{\tilde{c}\}$. Then we have $P_1|P_2 \xrightarrow{\tau} \nu\tilde{c}.(P_1'|P_2')$ and thus $\nu a.(P_1|P_2) \xrightarrow{\tau} \nu a, \tilde{c}.(P_1'|P_2')$. By definition of structural equivalence, we have $\nu\tilde{c}, a.(P_1'|P_2') \equiv \nu a, \tilde{c}.(P_1'|P_2')$.

Case REFLEXIVITY: $P \equiv P$

Immediately holds.

Case TRANSITIVITY: $P_1 \equiv P_3$

Assume $P_1 \equiv P_2$, $P_2 \equiv P_3$ and $P_1 \xrightarrow{\alpha} P_1'$. By the induction hypothesis, we have $P_2 \xrightarrow{\alpha} P_2'$, $P_3 \xrightarrow{\alpha} P_3'$, $P_1' \equiv P_2'$ and $P_2' \equiv P_3'$. Therefore $P_1' \equiv P_3'$ and of course $P_3 \xrightarrow{\alpha} P_3'$.

Case SYMMETRY: $Q \equiv P$

Assume $P \equiv Q$ and by clause 2 it holds that $Q \xrightarrow{\alpha} Q'$ implies $P \xrightarrow{\alpha} P'$ and $P' \equiv Q'$. We have $Q \xrightarrow{\alpha} Q'$ and therefore $P \xrightarrow{\alpha} P'$. By rule of symmetry and $P' \equiv Q'$, we have $Q' \equiv P'$.

Case EVALUATION CONTEXTS: $C[P] \equiv C[Q]$

Similar to the case of $P_1|(P_2|P_3) \equiv (P_1|P_2)|P_3$.

□

B.2 Soundness of environmental bisimulation up-to context

Lemma B.2 (input transition). *Let $P_1\mathcal{E}^*Q_1$ and $a\mathcal{E}b$. Suppose that $W_1 = b$ for any $a\hat{\mathcal{E}}W_1$. If $P_1 \xrightarrow{a(V)} P_1'$, then for any W, there exists some Q_1' such that $Q_1 \xrightarrow{b(W)} Q_1'$ with $P_1'(\mathcal{E} \cup \{(V, W)\})^*Q_1'$.*

Proof. By induction on transition derivation of $P_1 \xrightarrow{a(V)} P_1'$.

Case IN: $C = C_1(x).C_2$

The transition of P_1 must be of the form $C_1[\tilde{V}](x).C_2[\tilde{V}] \xrightarrow{a(V)} \{^V/_x\}C_2[\tilde{V}]$, where $eval(C_1[\tilde{V}]) = a$. Then we have $eval(C_1[\tilde{W}]) = b$. Therefore $Q_1 = C_1[\tilde{W}](x).C_2[\tilde{W}] \xrightarrow{b(W)} \{^W/_x\}C_2[\tilde{V}]$ and $\{^V/_x\}C_2[\tilde{V}](\mathcal{E} \cup \{(V, W)\})^*\{^W/_x\}C_2[\tilde{W}]$.

Case PAR-L: $C = C_1|C_2 \quad C_1[\tilde{V}] \xrightarrow{a(V)} P'_{11}$

Assume that $C_1[\tilde{V}]|C_2[\tilde{V}] \xrightarrow{a(V)} P'_{11}|C_2[\tilde{V}]$ holds. By the induction hypothesis and $a\mathcal{E}b$, there exists Q'_{11} such that $C_1[\tilde{W}] \xrightarrow{b(W)} Q'_{11}$ and $P'_{11}(\mathcal{E} \cup \{(V, W)\})^*Q'_{11}$. Therefore, $C_1[\tilde{W}]|C_2[\tilde{W}] \xrightarrow{b(W)} Q'_{11}|C_2[\tilde{W}]$. By $C_2[\tilde{V}]\mathcal{E}^*C_2[\tilde{W}]$, we have $C_2[\tilde{V}](\mathcal{E}\cup\{(V, W)\})^*C_2[\tilde{W}]$. Thus $P'_{11}|C_2[\tilde{V}](\mathcal{E}\cup\{(V, W)\})^*Q'_{11}|C_2[\tilde{W}]$.

Case PAR-R: $C = C_1|C_2 \quad C_2[\tilde{V}] \xrightarrow{a(V)} P'_{12}$

Similar to PAR-L.

Case REP: $C = !C_1 \quad !C_1[\tilde{V}] \xrightarrow{a(V)} P'_{11}$

We have $C_1[\tilde{V}]|!C_1[\tilde{V}] \xrightarrow{a(V)} P'_{11}$. By the induction hypothesis and $a\mathcal{E}b$, we have $C_1[\tilde{W}]|!C_1[\tilde{W}] \xrightarrow{b(W)} Q'_{11}$ and $P'_{11}(\mathcal{E} \cup \{(V, W)\})^*Q'_{11}$ for some Q'_{11}. Therefore $!C_1[\tilde{W}] \xrightarrow{b(W)} Q'_{11}$ and of course $P'_{11}(\mathcal{E} \cup \{(V, W)\})^*Q'_{11}$.

Case SCOPE: $C = \nu c.C_1 \quad \nu c.C_1[\tilde{V}] \xrightarrow{a(V)} \nu c.P'_{11}$

Assume that $C_1[\tilde{V}] = C'_1[\tilde{V}, c]$ holds. Then we have $C'_1[\tilde{V}, c] \xrightarrow{a(V)} P'_{11}$. By the induction hypothesis, we have $C'_1[\tilde{W}, c] \xrightarrow{b(W)} Q'_{11}$ and $P'_{11}(\mathcal{E} \cup \{(c, c)\} \cup \{(V, W)\})^*Q'_{11}$ for some Q'_{11}. Then we have $\nu c.C_1[\tilde{W}] \xrightarrow{b(W)} \nu c.Q'_{11}$. By $P'_{11}(\mathcal{E} \cup \{(c, c)\} \cup \{(V, W)\})^*Q'_{11}$, we have $\nu c.P'_{11}(\mathcal{E} \cup \{(V, W)\})^*\nu c.Q'_{11}$. □

Lemma B.3 (output transition). *Let $P_1\mathcal{E}^*Q_1$ and $a\mathcal{E}b$. Suppose that $W_1 = b$ for any $a\hat{\mathcal{E}}W_1$. If $P_1 \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P'_1$ with $\tilde{c} \notin \mathsf{fn}(\#_1(\mathcal{E}))$, then there exist some Q'_1, W and $\tilde{d}$ with $V(\widehat{\mathcal{E} \cup \{(\tilde{c}, \tilde{d})\}})^*W$ such that $Q_1 \xrightarrow{\nu\tilde{d}.\overline{b}\langle W\rangle} Q'_1$ with $\tilde{d} \notin \mathsf{fn}(\#_2(\mathcal{E}))$ and $P'_1(\mathcal{E} \cup \{(\tilde{c}, \tilde{d})\})^*Q'_1$.*

Proof. By induction on transition derivation of $P_1 = C[\tilde{V}] \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P'_1$.

Case OUT: $C = \overline{C_1}\langle C_2\rangle.C_3$

We have $\overline{C_1[\tilde{V}]}\langle C_2[\tilde{V}]\rangle.C_3[\tilde{V}] \xrightarrow{\overline{a}\langle V\rangle} C_3[\tilde{V}]$ where $eval(C_1[\tilde{V}]) = a$ and $eval(C_2[\tilde{V}]) = V$. By $a\mathcal{E}b$, we have $eval(C_1[\tilde{W}]) = b$. Therefore $\overline{C_1[\tilde{W}]}\langle C_2[\tilde{W}]\rangle.C_3[\tilde{W}] \xrightarrow{\overline{b}\langle W\rangle} C_3[\tilde{W}]$, $W = eval(C_2[\tilde{W}])$ and $C_3[\tilde{V}]\mathcal{E}^*C_3[\tilde{W}]$.

Case PAR-L: $C = C_1|C_2 \quad C_1[\tilde{V}] \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P'_{11} \quad \tilde{c} \notin \mathsf{fn}(C_2[\tilde{V}])$

Assume that $C_1[\tilde{V}]|C_2[\tilde{V}] \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P'_{11}|C_2[\tilde{V}]$ holds. By the induction hypothesis, we have $C_1[\tilde{W}] \xrightarrow{\nu\tilde{d}.\overline{b}\langle W\rangle} Q'_{11}$, $P'_{11}(\mathcal{E}\cup\{(\tilde{c}, \tilde{d})\})^*Q'_{11}$, $V(\mathcal{E}\cup\{(\tilde{c}, \tilde{d})\})^*W$ and $\tilde{d} \notin \mathsf{fn}(\#_2(\mathcal{E}))$ for some $Q'_{11}, \tilde{d}$. By $\tilde{d} \notin \mathsf{fn}(C_2[\tilde{W}])$, we have $C_1[\tilde{W}]|C_2[\tilde{W}] \xrightarrow{\nu\tilde{d}.\overline{b}\langle W\rangle} Q'_{11}|C_2[\tilde{W}]$. By $P'_{11}(\mathcal{E} \cup \{(\tilde{c}, \tilde{d})\})^*Q'_{11}$, we have

$P'_{11}|C_2[\tilde{V}](\mathcal{E} \cup \{(\tilde{c}, \tilde{d})\})^* Q'_{11}|C_2[\tilde{W}]$.

Case PAR-R: $C = C_1|C_2 \quad C_2[\tilde{V}] \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P'_{12} \quad \tilde{c} \notin \mathsf{fn}(C_1[\tilde{V}])$
Similar to PAR-L.

Case REP: $C = !C_1 \quad !C_1[\tilde{V}] \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P'_{11}$
We have $C_1[\tilde{V}]|!C_1[\tilde{V}] \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P'_{11}$. By the induction hypothesis, we have $C_1[\tilde{W}]|!C_1[\tilde{W}] \xrightarrow{\nu\tilde{d}.\overline{b}\langle W\rangle} Q'_{11}$, $P'_{11}(\mathcal{E} \cup \{(\tilde{c}, \tilde{d})\})^* Q'_{11}$, $V(\mathcal{E} \cup \{(\tilde{c}, \tilde{d})\})^* W$ and $\tilde{d} \notin \mathsf{fn}(\#_2(\mathcal{E}))$ for some $Q'_{11}, \tilde{d}$. Then we have $!C_1[\tilde{W}] \xrightarrow{\nu\tilde{d}.\overline{b}\langle W\rangle} Q'_{11}$ and of course $P'_{11}(\mathcal{E} \cup \{(\tilde{c}, \tilde{d})\})^* Q'_{11}$

Case SCOPE: $C = \nu c.C_1 \quad \nu c.C_1[\tilde{V}] \xrightarrow{\nu\tilde{c_1}.\overline{a}\langle V\rangle} \nu c.P'_{11}$
Assume that $C_1[\tilde{V}] = C'_1[\tilde{V}, c]$ holds. By SCOPE, we have $C'_1[\tilde{V}, c] \xrightarrow{\nu\tilde{c_1}.\overline{a}\langle V\rangle} P'_{11}$ and $c \notin \mathsf{n}(\nu\tilde{c_1}.\overline{a}\langle V\rangle)$. By the induction hypothesis, we have $C'_1[\tilde{W}, c] \xrightarrow{\nu\tilde{d_1}.\overline{b}\langle W\rangle} Q'_{11}$, $P'_{11}(\mathcal{E} \cup \{(c,c)\} \cup \{(\tilde{c_1}, \tilde{d_1})\})^* Q'_{11}$, $V(\mathcal{E} \cup \{(\tilde{c_1}, \tilde{d_1}), (c,c)\})^* W$, $\tilde{d_1} \notin \mathsf{fn}(\#_2(\mathcal{E} \cup \{(c,c)\}))$ and $c \notin \mathsf{n}(\nu\tilde{d_1}.\overline{b}\langle W\rangle)$ for some $Q'_{11}, \tilde{d_1}$. Therefore $\nu c.C_1[\tilde{W}] \xrightarrow{\nu\tilde{d_1}.\overline{b}\langle W\rangle} \nu c.Q'_{11}$ and $\nu c.P'_{11}(\mathcal{E} \cup \{(\tilde{c_1}, \tilde{d_1})\})^* \nu c.Q'_{11}$.

Case OPEN: $C = \nu c.C_1 \quad \nu c.C_1[\tilde{V}] \xrightarrow{\nu\tilde{c_1}, c.\overline{a}\langle V\rangle} P'_{11}$
Assume that $C_1[\tilde{V}] = C'_1[\tilde{V}, c]$ holds. By OPEN, We have $C'_1[\tilde{V}, c] \xrightarrow{\nu\tilde{c_1}.\overline{a}\langle V\rangle} P'_{11}$, $c \neq a$ and $c \in \mathsf{fn}(V) \setminus \{\tilde{c_1}\}$. By the induction hypothesis, we have $C'_1[\tilde{W}, c] \xrightarrow{\nu\tilde{d_1}.\overline{b}\langle W\rangle} Q_{11}$, $P'_{11}(\mathcal{E} \cup \{(c,c)\} \cup \{(\tilde{c_1}, \tilde{d_1})\})^* Q'_{11}$, $V(\mathcal{E} \cup \{(\tilde{c_1}, \tilde{d_1}), (c,c)\})^* W$, $\tilde{d_1} \notin \mathsf{fn}(\#_2(\mathcal{E} \cup \{(c,c)\}))$, $c \neq b$ and $c \in \mathsf{fn}(W) \setminus \{\tilde{d_1}\}$ for some $Q'_{11}, C_2, \tilde{d_1}$. Therefore $\nu c.C_1[\tilde{W}] \xrightarrow{\nu\tilde{d_1}, c.\overline{b}\langle w\rangle} Q'_{11}$ and $Q'_{11}(\mathcal{E} \cup \{(c,c)\} \cup \{(\tilde{c_1}, \tilde{d_1})\})^* Q'_{11}$.

□

Lemma B.4 (τ transition). *Suppose $P_0 \mathcal{Y}_{\mathcal{E}} Q_0$ and $P_1 \hat{\mathcal{E}}^* Q_1$ for an environmental bisimulation $\mathcal{Y}$ up-to context. If $P_1 \xrightarrow{\tau} P'_1$, then there exists some Q'_1 such that $Q_1 \xrightarrow{\tau} Q'_1$ with $P_0|P'_1 \mathcal{Y}^{(*)}_{\mathcal{E}} Q_0|Q'_1$.*

Proof. By induction on transition derivation of $P_1 = C[\tilde{V}] \xrightarrow{\tau} P'_1$.

Case PAR-L: $C = C_1|C_2 \quad C_1[\tilde{V}] \xrightarrow{\tau} P'_{11}$
Assume that $C_1[\tilde{V}]|C_2[\tilde{V}] \xrightarrow{\tau} P'_{11}|C_2[\tilde{V}]$ holds. By the induction hypothesis, we have $C_1[\tilde{W}] \xrightarrow{\tau} Q'_{11}$ and $P_0|P'_{11} \mathcal{Y}^{(*)}_{\mathcal{E}} Q_0|Q'_{11}$ for some Q'_{11}. Therefore $C_1[\tilde{W}]|C_2[\tilde{W}] \xrightarrow{\tau} Q'_{11}|C_2[\tilde{W}]$. By $C_2[\tilde{V}]\mathcal{E}^* C_2[\tilde{W}]$, we have $P_0|P'_{11}|C_2[\tilde{V}] \mathcal{Y}^{(*)}_{\mathcal{E}} Q_0|Q'_1|C_2[\tilde{W}]$.

Case PAR-R: $C = C_1|C_2 \quad C_2[\tilde{V}] \xrightarrow{\tau} P'_{12}$
Similar to PAR-L.

Case TAU-L: $C = C_1|C_2 \quad C_1[\tilde{V}] \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P'_{11} \quad C_2[\tilde{V}] \xrightarrow{a(V)} P'_{12}$
Assume that $C_1[\tilde{V}]|C_2[\tilde{V}] \xrightarrow{\tau} \nu\tilde{c}.(P'_{11}|P'_{12})$ holds. We can choose fresh $\tilde{c}$ (i.e. $\tilde{c} \notin \mathsf{fn}(P_0, \#_1(\mathcal{E}))$). By Lemma B.3, we have $C_1[\tilde{W}] \xrightarrow{\nu\tilde{d}.\overline{b}\langle W\rangle} Q'_{11}$ and $P'_{11}(\hat{\mathcal{E}} \cup \{(\tilde{c}, \tilde{d})\})^* Q'_{11}$ for some Q'_{11} and $\tilde{d} \notin \mathsf{fn}(Q_0, \#_2(\mathcal{E}))$, and we also have $V \widehat{\mathcal{E} \cup \{(\tilde{c}, \tilde{d})\}}^* W$. By Lemma B.2, we have $C_2[\tilde{W}] \xrightarrow{b(W)} Q'_{12}$ and $P'_{12}(\hat{\mathcal{E}} \cup \{(V, W)\})^* Q'_{12}$ for some Q'_{12}. Hence $P'_{11} \widehat{\mathcal{E} \cup \{(\tilde{c}, \tilde{d})\}}^* Q'_{11}$, $P'_{12} \widehat{\mathcal{E} \cup \{(\tilde{c}, \tilde{d})\}}^* Q'_{12}$ and $C_1[\tilde{W}]|C_2[\tilde{W}]$

$\xrightarrow{\tau} \nu\tilde{d}.(Q'_{11}|Q'_{12})$. By $\tilde{d} \notin \mathsf{fn}(Q_0, \#_2(\mathcal{E}))$ and clause 7, we have $P_0 \mathcal{Y}_{\mathcal{E}\cup\{(\tilde{c},\tilde{d})\}} Q_0$. Therefore $\nu\tilde{c}.(P_0|P'_{11}|P'_{12}) \mathcal{Y}^{(*)}_{\mathcal{E}} \nu\tilde{d}.(Q_0|Q'_{11}|Q'_{12})$.

Case TAU-R: $C = C_1|C_2 \quad C_1[\tilde{V}] \xrightarrow{a(V)} P'_{11} \quad C_2[\tilde{V}] \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P'_{12}$
Similar to TAU-L.

Case REP: $C = !C_1 \quad !C_1[\tilde{V}] \xrightarrow{\tau} P'_1$
We have $C_1[\tilde{V}]||!C_1[\tilde{V}] \xrightarrow{\tau} P'_1$. By the induction hypothesis and $a\mathcal{E}b$, we have $C_1[\tilde{W}]||!C_1[\tilde{W}] \xrightarrow{\tau} Q'_1$ and $P_0|P'_1\mathcal{Y}^{(*)}_{\mathcal{E}}Q_0|Q'_1$ for some Q'_1. Therefore $!C_1[\tilde{W}] \xrightarrow{\tau} Q'_1$ and of course $P_0|P'_1\mathcal{Y}^{(*)}_{\mathcal{E}}Q_0|Q'_1$.

Case SCOPE: $C = \nu c.C_1 \quad \nu c.C_1[\tilde{V}] \xrightarrow{\tau} \nu c.P'_{11}$.
We have $C_1[\tilde{V}] \xrightarrow{\tau} P'_{11}$. Assume that $C_1[\tilde{V}] = C'_1[\tilde{V}, c]$ holds, where $\mathsf{fn}(C'_1) = \mathsf{bn}(C'_1) \cap \mathsf{fn}(\mathcal{E} \cup \{(c,c)\}, P_0, Q_0) = \emptyset$. By the induction hypothesis, we have $C'_1[\tilde{W}, c] \xrightarrow{\tau} Q'_{11}$ and $P_0|P'_{11}\mathcal{Y}^{(*)}_{\mathcal{E}\cup\{(c,c)\}}Q_0|Q'_{11}$ for some Q'_{11}. Therefore $\nu c.C_1[\tilde{W}] \xrightarrow{\tau} \nu c.Q'_{11}$. By $P_0|\nu c.P'_{11} \equiv \nu c.(P_0|P'_{11})$, $Q_0|\nu c.Q'_{11} \equiv \nu c.(Q_0|Q'_{11})$ and $\hat{\mathcal{E}} \subseteq \{(V,W) \mid V\,\widehat{\mathcal{E}\cup\{(c,c)\}}^{*}\,W \mathit{\ and\ } c \notin \mathsf{fn}(V,W)\}$, we have $P_0|\nu c.P'_{11}\mathcal{Y}^{(*)}_{\mathcal{E}}Q_0|\nu c.Q'_{11}$.

Case RUN: $C = \mathit{run}(C_1)$
Suppose $\mathit{run}(C_1[\tilde{V}]) \xrightarrow{\tau} P'_1$, where $\mathit{eval}(C_1[\tilde{V}]) = {}^{\backprime}P'_1$. By definition of terms and $C_1[\tilde{V}]\hat{\mathcal{E}}^*C_1[\tilde{W}]$, we have $\mathit{eval}(C_1[\tilde{V}]) = C'_1[\tilde{V}] = {}^{\backprime}P'_1$, $\mathit{eval}(C_1[\tilde{W}]) = C'_1[\tilde{W}]$ for some C'_1. Now there are two subcases of C'_1.

Subcase: $C'_1 = []$
We have $\mathit{eval}(C_1[\tilde{V}]) = {}^{\backprime}P'_1\hat{\mathcal{E}}N = \mathit{eval}(C_1[\tilde{W}])$ for some N. By clause 5, 6, and 8, there exists Q'_1 such that $N = {}^{\backprime}Q'_1$ and $P_0|P'_1\mathcal{Y}^{(*)}_{\mathcal{E}}Q_0|Q'_1$. Therefore $\mathit{run}(C_1[\tilde{W}]) \xrightarrow{\tau} Q'_1$ and $P_0|P'_1\mathcal{Y}^{(*)}_{\mathcal{E}}Q_0|Q'_1$.

Subcase: $C'_1 = {}^{\backprime}C'_{11}$
We have $\mathit{eval}(C_1[\tilde{V}]) = {}^{\backprime}C'_{11}[\tilde{V}]\hat{\mathcal{E}}^*{}^{\backprime}C'_{11}[\tilde{W}] = \mathit{eval}(C_1[\tilde{W}])$. Therefore $\mathit{run}(C_1[\tilde{V}]) \xrightarrow{\tau} C'_{11}[\tilde{V}]$ and $\mathit{run}(C_1[\tilde{W}]) \xrightarrow{\tau} C'_{11}[\tilde{W}]$. By $C'_{11}[\tilde{V}]\hat{\mathcal{E}}^*C'_{11}[\tilde{W}]$, we have $P_0|C'_{11}[\tilde{V}]\mathcal{Y}^{(*)}_{\mathcal{E}}Q_0|C'_{11}[\tilde{W}]$.

Case IFTRUE: $C = \mathit{if}\ C_1 = C_2\ \mathit{then}\ C_3\ \mathit{else}\ C_4 \quad \mathit{eval}(C_1[\tilde{V}]) = \mathit{eval}(C_2[\tilde{V}])$
We have $\mathit{if}\ C_1[\tilde{V}] = C_2[\tilde{V}]\ \mathit{then}\ C_3[\tilde{V}]\ \mathit{else}\ C_4[\tilde{V}] \xrightarrow{\tau} C_3[\tilde{V}]$. By clause 5 and 8, we have $\mathit{eval}(C_1[\tilde{W}]) = \mathit{eval}(C_2[\tilde{W}])$. Therefore $\mathit{if}\ C_1[\tilde{W}] = C_2[\tilde{W}]\ \mathit{then}\ C_3[\tilde{W}]\ \mathit{else}\ C_4[\tilde{W}] \xrightarrow{\tau} C_3[\tilde{W}]$. By $C_3[\tilde{V}]\hat{\mathcal{E}}^*C_3[\tilde{W}]$, we have $P_0|C_3[\tilde{V}]\mathcal{Y}^{(*)}_{\mathcal{E}}Q_0|C_3[\tilde{W}]$.

Case IFFALSE: $C = \mathit{if}\ C_1 = C_2\ \mathit{then}\ C_3\ \mathit{else}\ C_4 \quad \mathit{eval}(C_1[\tilde{V}]) \neq \mathit{eval}(C_2[\tilde{V}])$
Similar to IFTRUE.

Case MATCH: $C = \mathit{match}\ C_1\ \mathit{as}\ x\ \mathit{in}\ C_2 \quad \mathit{eval}(C_1[\tilde{V}]) = C'_1[\tilde{V}] \in \mathbf{Quo}$
There are 3 subcases of C'_1.

Subcase: $C'_1 = [] \quad \mathit{match}\ C_1[\tilde{V}]\ \mathit{as}\ x\ \mathit{in}\ C_2[\tilde{V}] \xrightarrow{\tau} \nu b.\{^{\mathit{reify}_b(V)}/_x\}C_2[\tilde{V}]$
We can suppose $b \notin \mathsf{fn}(P_0, Q_0, V, W, C_2[\tilde{V}], C_2[\tilde{W}])$. Suppose $V\hat{\mathcal{E}}W$ in the hole of C'_1. By clause 8d, we have $\exists b \notin \mathsf{fn}(\mathcal{E}).P_0\mathcal{Y}_{\mathcal{E}\cup\{(\mathit{reify}_b(V),\mathit{reify}_b(W))\}}Q_0$. Then we have $\mathit{match}\ C_1[\tilde{W}]\ \mathit{as}\ x\ \mathit{in}\ C_2[\tilde{W}] \xrightarrow{\tau} \nu b.\{^{\mathit{reify}_b(W)}/_x\}C_2[\tilde{W}]$ by clause 5. Since $\{^{\mathit{reify}_b(V)}/_x\}C_2[\tilde{V}]\hat{\mathcal{E}'}^{*}\{^{\mathit{reify}_b(W)}/_x\}C_2[\tilde{W}]$ for $\mathcal{E}' = \mathcal{E} \cup \{(\mathit{reify}_b(V), \mathit{reify}_b(W))\}$, we have

$P_0|\nu b.\{^{reify_b(V)}/_x\}C_2[\tilde{V}] \equiv \nu b.(P_0|\{^{reify_b(V)}/_x\}C_2[\tilde{V}])\mathcal{Y}^{(*)}_{\mathcal{E}}\nu b.(Q_0|\{^{reify_b(W)}/_x\}C_2[\tilde{W}])$
$\equiv Q_0|\nu b.\{^{reify_b(W)}/_x\}C_2[\tilde{W}]$. Therefore $P_0|\nu b.\{^{reify_b(V)}/_x\}C_2[\tilde{V}]\mathcal{Y}^{(*)}_{\mathcal{E}}Q_0|\nu b.\{^{reify_b(W)}/_x\}C_2[\tilde{W}]$.

Subcase: $C_1' = `[] \quad match\, C_1[\tilde{V}]\, as\, x\, in\, C_2[\tilde{V}] \xrightarrow{\tau} \nu b.\{^{reify_b(`V)}/_x\}C_2[\tilde{V}]$

Assume that $V\hat{\mathcal{E}}W$ holds in the hole of C_1' and $b \notin \mathsf{fn}(P_0, Q_0, `V, `W, C_2[\tilde{V}], C_2[\tilde{W}])$ holds. We have $`W \in$ **Quo**. By MATCH, we have $match\, C_1[\tilde{W}]\, as\, x\, in\, C_2[\tilde{W}] \xrightarrow{\tau} \nu b.\{^{reify_b(`W)}/_x\}C_2[\tilde{W}]$. By $(reify(`V), reify(`W)) \in \hat{\mathcal{E}}$, we have $\{^{reify(`V)}/_x\}C_2[\tilde{V}]\hat{\mathcal{E}}^*\{^{reify_b(`W)}/_x\}C_2[\tilde{W}]$. Then we have $P_0|\nu b.\{^{reify(`V)}/_x\}C_2[\tilde{V}] \equiv \nu b.(P_0|\{^{reify(`V)}/_x\}C_2[\tilde{V}])\mathcal{Y}^{(*)}_{\mathcal{E}}\nu b.(Q_0|\{^{reify(`W)}/_x\}C_2[\tilde{W}]) \equiv Q_0|\nu b.\{^{reify(`W)}/_x\}C_2[\tilde{W}]$. Therefore $P_0|\nu b.\{^{reify(`V)}/_x\}C_2[\tilde{V}]\mathcal{Y}^{(*)}_{\mathcal{E}}Q_0|\nu b.\{^{reify(`W)}/_x\}C_2[\tilde{W}]$.

Subcase: Otherwise $match\, C_1[\tilde{V}]\, as\, x\, in\, C_2[\tilde{V}] \xrightarrow{\tau} \nu b.\{^{reify_b(C_1'[\tilde{V}])}/_x\}C_2[\tilde{V}]$

We can assume that $b \notin \mathsf{fn}(P_0, Q_0, C_1'[\tilde{V}], C_1'[\tilde{W}], C_2[\tilde{V}], C_2[\tilde{W}])$ holds. By clause 8d, we have $\exists b \notin \mathsf{fn}(\mathcal{E}).P_0\mathcal{Y}_{\mathcal{E}\cup\{(reify_b(C_1'[\tilde{V}]), reify_b(C_1'[\tilde{W}]))\}}Q_0$. We have $C_1'[\tilde{W}] \in$ **Quo** by clause 5, thus by MATCH, we have $match\, C_1[\tilde{W}]\, as\, x\, in\, C_2[\tilde{W}] \xrightarrow{\tau} \nu b.\{^{reify_b(C_1'[\tilde{W}])}/_x\}C_2[\tilde{W}]$. By $\{^{reify_b(C_1'[\tilde{V}])}/_x\}C_2[\tilde{V}]\hat{\mathcal{E}'}^*\{^{reify_b(C_1'[\tilde{W}])}/_x\}C_2[\tilde{W}]$ for $\mathcal{E}' = \mathcal{E} \cup \{(reify_b(C_1'[\tilde{V}]), reify_b(C_1'[\tilde{W}]))\}$, we have $P_0|\nu b.\{^{reify_b(C_1'[\tilde{V}])}/_x\}C_2[\tilde{V}] \equiv \nu b.(P_0|\{^{reify_b(C_1'[\tilde{V}])}/_x\}C_2[\tilde{V}])\mathcal{Y}^{(*)}_{\mathcal{E}}\ \nu b.(Q_0|\{^{reify_b(C_1'[\tilde{W}])}/_x\}C_2[\tilde{W}]) \equiv Q_0|\nu b.\{^{reify_b(C_1'[\tilde{W}])}/_x\}C_2[\tilde{W}]$. Therefore $P_0|\nu b.\{^{reify_b(C_1'[\tilde{V}])}/_x\}C_2[\tilde{V}]\mathcal{Y}^{(*)}_{\mathcal{E}}Q_0|\nu b.\{^{reify_b(C_1'[\tilde{W}])}/_x\}C_2[\tilde{W}]$. □

Theorem B.1 (soundness of environmental bisimulation up-to context). *Let $\mathcal{Y}$ be the environmental bisimilarity up-to context. Then $\mathcal{X} = \{(\mathcal{E}, P, Q) \mid P\mathcal{Y}^{(*)}_{\mathcal{E}}Q\}$ is an environmental bisimulation.*

Proof. Suppose $P\mathcal{X}_{\mathcal{E}}Q$, i.e.,$P\mathcal{Y}^{(*)}_{\mathcal{E}}Q$. Therefore for some P_0, P_1, Q_0, Q_1, $\mathcal{E}'$, $\tilde{c}$, and $\tilde{d}$, we have $\tilde{c} \notin \mathsf{fn}(\#_1(\mathcal{E}))$, $\tilde{d} \notin \mathsf{fn}(\#_2(\mathcal{E}))$, $P \equiv \nu\tilde{c}.(P_0|P_1)$, $Q \equiv \nu\tilde{d}.(Q_0|Q_1)$, $\hat{\mathcal{E}} \subseteq \{(V, W) \mid V\hat{\mathcal{E}'}^*W\, and\, \mathsf{fn}(V) \cap \{\tilde{c}\} = \mathsf{fn}(W) \cap \{\tilde{d}\} = \emptyset\}$, $P_0\mathcal{Y}_{\mathcal{E}'}Q_0$ and $P_1\hat{\mathcal{E}'}^*Q_1$. We are going to show the 9 clauses hold.

Case 1: $P \xrightarrow{\tau} P'$

By Lemma B.1, we have 4 subcases of the transition of $\nu\tilde{c}.(P_0|P_1)$.

Subcase: $P_0 \xrightarrow{\tau} P_0'$

By $P_0\mathcal{Y}_{\mathcal{E}'}Q_0$, we have $Q_0 \xrightarrow{\tau} \ldots \xrightarrow{\tau} Q_0'$ and $P_0'\mathcal{Y}^{(*)}_{\mathcal{E}'}Q_0'$. Therefore $\nu\tilde{c}.(P_0|P_1) \xrightarrow{\tau} \nu\tilde{c}.(P_0'|P_1) \equiv \nu\tilde{c}, \tilde{c}_1.(P_{00}'|P_{01}'|P_1) \equiv P'$ and $\nu\tilde{d}.(Q_0|Q_1) \xrightarrow{\tau} \ldots \xrightarrow{\tau} \nu\tilde{d}.(Q_0'|Q_1) \equiv \nu\tilde{d}, \tilde{d}_1.(Q_{00}'|Q_{01}'|Q_1) \equiv Q'$. Hence $P'\mathcal{Y}^{(*)}_{\mathcal{E}}Q'$, i.e., $P'\mathcal{X}_{\mathcal{E}}Q'$.

Subcase: $P_1 \xrightarrow{\tau} P_1'$

By Lemma B.4, we have $Q_1 \xrightarrow{\tau} Q_1'$ and $P_0|P_1'\mathcal{Y}^{(*)}_{\mathcal{E}'}Q_0|Q_1'$. Therefore $\nu\tilde{c}.(P_0|P_1) \xrightarrow{\tau} \nu\tilde{c}.(P_0|P_1') \equiv P'$ and $\nu\tilde{d}.(Q_0|Q_1) \xrightarrow{\tau} \nu\tilde{d}.(Q_0|Q_1') \equiv Q'$. Hence $P'\mathcal{Y}^{(*)}_{\mathcal{E}}Q'$, i.e., $P'\mathcal{X}_{\mathcal{E}}Q'$.

Subcase: $P_0 \xrightarrow{\nu\tilde{c_1}.\overline{a}\langle V\rangle} P_0' \quad P_1 \xrightarrow{a(V)} P_1'$

By $P_0\mathcal{Y}_{\mathcal{E}'}Q_0$, we have $Q_0 \xrightarrow{\tau} \ldots \xrightarrow{\nu\tilde{d_1}.\overline{b}\langle W\rangle} \ldots \xrightarrow{\tau} Q_0'$ and $P_0'\mathcal{Y}_{\mathcal{E}'\cup\{(V,W)\}}Q_0'$, while $\tilde{d}_1 \notin \mathsf{fn}(\#_2(\mathcal{E}'))$. Then there exists just one b such that $a\hat{\mathcal{E}}b$ by the definition of terms, and by Lemma B.2, we have $Q_1 \xrightarrow{b(W)} Q_1'$ and $P_1'(\hat{\mathcal{E}'} \cup \{(V, W)\})^*Q_1'$. Therefore $\nu\tilde{c}.(P_0|P_1) \xrightarrow{\tau} \nu\tilde{c}, \tilde{c}_1.(P_0'|P_1') \equiv P'$ and $\nu\tilde{d}.(Q_0|Q_1) \xrightarrow{\tau} \nu\tilde{d}, \tilde{d}_1.(Q_0'|Q_1') \equiv Q'$. Hence $P'\mathcal{Y}^{(*)}_{\mathcal{E}}Q'$, i.e., $P'\mathcal{X}_{\mathcal{E}}Q'$.

Subcase: $P_0 \xrightarrow{a(V)} P_0' \quad P_1 \xrightarrow{\nu\tilde{c_1}.\overline{a}\langle V\rangle} P_1'$

There exists just one b such that $a\hat{\mathcal{E}}b$ by the definition of terms, and by Lemma B.3, we have $Q_1 \xrightarrow{\nu\tilde{d_1}.\overline{b}\langle W\rangle} Q_1'$ and $P_1'(\hat{\mathcal{E}'} \cup \{(\tilde{c}_1, \tilde{d}_1)\})^*Q_1'$. We also have $V\,\widehat{\mathcal{E}' \cup \{(\tilde{c}_1, \tilde{d}_1)\}}^*\,W$ by Lemma B.3.

Now we have $P_0 \mathcal{Y}_{\mathcal{E}' \cup \{(\tilde{c_1}, \tilde{d_1})\}} Q_0$ by $\tilde{d}_1 \notin \mathsf{fn}(\#_2(\mathcal{E}))$ and clause 7. Therefore $Q_0 \xrightarrow{\tau} \ldots \xrightarrow{b(W)} \ldots \xrightarrow{\tau} Q_0'$ and $P_0' \mathcal{Y}^{(*)}_{\mathcal{E}' \cup \{(\tilde{c_1}, \tilde{d_1})\}} Q_0'$. Then it holds that $\nu\tilde{c}.(P_0|P_1) \xrightarrow{\tau} \nu\tilde{c}, \tilde{c}_1.(P_0'|P_1') \equiv P'$ and $\nu\tilde{d}.(Q_0|Q_1) \xrightarrow{\tau} \nu\tilde{d}, \tilde{d}_1.(Q_0'|Q_1') \equiv Q'$. Hence $P' \mathcal{Y}^{(*)}_{\mathcal{E}} Q'$, i.e., $P' \mathcal{X}_{\mathcal{E}} Q'$.

Case 2: $P \xrightarrow{a(V)} P' \quad a\hat{\mathcal{E}}b \quad V\hat{\mathcal{E}}^* W$

By Lemma B.1, we have 2 subcases of the transition of $\nu\tilde{c}.(P_0|P_1)$.

Subcase: $P_0 \xrightarrow{a(V)} P_0'$

By $P_0 \mathcal{Y}_{\mathcal{E}'} Q_0$, we have $Q_0 \xrightarrow{\tau} \ldots \xrightarrow{b(W)} \ldots \xrightarrow{\tau} Q_0'$ and $P_0' \mathcal{Y}^{(*)}_{\mathcal{E}'} Q_0'$ for some Q_0'. Then it holds that $\nu\tilde{c}.(P_0|P_1) \xrightarrow{\tau} \nu\tilde{c}.(\nu\tilde{c}_1.(P_{00}'|P_{01}')|P_1) \equiv \nu\tilde{c}, \tilde{c}_1.(P_{00}'|P_{01}'|P_1) \equiv P'$ where $\tilde{c} \notin \mathsf{n}(a(V))$, and $\nu\tilde{d}.(Q_0|Q_1) \xrightarrow{\tau} \ldots \xrightarrow{\tau} \nu\tilde{d}.(\nu\tilde{d}_1.(Q_{00}'|Q_{01}')|Q_1) \equiv \nu\tilde{d}, \tilde{d}_1.(Q_{00}'|Q_{01}'|Q_1) \equiv Q'$ where $\tilde{d} \notin \mathsf{n}(b(W))$. Hence $P' \mathcal{Y}^{(*)}_{\mathcal{E}} Q'$, i.e., $P' \mathcal{X}_{\mathcal{E}} Q'$.

Subcase: $P_1 \xrightarrow{a(V)} P_1'$

By Lemma B.2, we have $Q_1 \xrightarrow{b(W)} Q_1'$ and $P_1' (\hat{\mathcal{E}'} \cup \{(V,W)\})^* Q_1'$ for some Q_1'. Therefore $\nu\tilde{c}.(P_0|P_1) \xrightarrow{a(V)} \nu\tilde{c}.(P_0|P_1') \equiv P'$ and $\nu\tilde{d}.(Q_0|Q_1) \xrightarrow{b(W)} \nu\tilde{d}.(Q_0|Q_1') \equiv Q'$. By $(\mathcal{E}' \cup \widehat{\{(V,W)\}})^* \subseteq \hat{\mathcal{E}'}^*$, we have $P' \mathcal{Y}^{(*)}_{\mathcal{E}} Q'$, i.e., $P' \mathcal{X}_{\mathcal{E}} Q'$.

Case 3: $P \xrightarrow{\nu\tilde{c_1}.\overline{a}\langle V \rangle} P' \quad a\hat{\mathcal{E}}b \quad \tilde{c_1} \notin \mathsf{fn}(\#_1(\mathcal{E}))$

By Lemma B.1, we have 2 subcases of the transition of $\nu\tilde{c}.(P_0|P_1)$.

Subcase: $P_0 \xrightarrow{\nu\tilde{c_2}.\overline{a}\langle V \rangle} P_0'$

We can assume that we have $\{\tilde{c_2}\} = \{\tilde{c_1}\} \setminus \{\tilde{c}\}$, $\{\tilde{c_3}\} = \{\tilde{c}\} \setminus \{\tilde{c_1}\}$, and $\tilde{c_2} \notin \mathsf{fn}(\#_1(\hat{\mathcal{E}'}))$. By $P_0 \mathcal{Y}_{\mathcal{E}'} Q_0$, we have $Q_0 \xrightarrow{\nu\tilde{d_2}.\overline{b}\langle W \rangle} Q_0'$ and $P_0' \mathcal{Y}^{(*)}_{\mathcal{E}' \cup \{(V,W)\}} Q_0'$ for some Q_0'. Therefore $\nu\tilde{c}.(P_0|P_1) \xrightarrow{\nu\tilde{c_1}.\overline{a}\langle V \rangle} \nu\tilde{c_3}.(P_0'|P_1) \equiv \nu\tilde{c_3}.(\nu\tilde{c_4}.(P_{00}'|P_{01})|P_1) \equiv P'$ and $\nu\tilde{d}.(Q_0|Q_1) \xrightarrow{\nu\tilde{d_1}.\overline{b}\langle W \rangle} \nu\tilde{d_3}.(Q_0'|Q_1) \equiv \nu\tilde{d_3}.(\nu\tilde{d_4}.(Q_{00}'|Q_{01})|Q_1) \equiv Q'$ where $\{\tilde{d_2}\} = \{\tilde{d_1}\} \setminus \{\tilde{d}\}$ and $\{\tilde{d_3}\} = \{\tilde{d}\} \setminus \{\tilde{d_1}\}$. Hence $P' \mathcal{Y}^{(*)}_{\mathcal{E} \cup \{(V,W)\}} Q'$, i.e., $P' \mathcal{X}_{\mathcal{E} \cup \{(V,W)\}} Q'$.

Subcase: $P_1 \xrightarrow{\nu\tilde{c_2}.\overline{a}\langle V \rangle} P_1'$

We can assume that we have $\{\tilde{c_2}\} = \{\tilde{c_1}\} \setminus \{\tilde{c}\}$ and $\tilde{c_2} \notin \mathsf{fn}(\#_1(\hat{\mathcal{E}'}))$. Now there exist Q_1' and $\tilde{d_2}$ such that $Q_1 \xrightarrow{\nu\tilde{d_2}.\overline{b}\langle W \rangle} Q_1'$, $P_1' (\hat{\mathcal{E}'} \cup \{(\tilde{c_2}, \tilde{d_2})\})^* Q_1'$, $V = (\mathcal{E}' \cup \widehat{\{(\tilde{c_2}, \tilde{d_2})\}})^* W$ and $\tilde{d_2} \notin \mathsf{fn}(\#_2(\hat{\mathcal{E}'}))$ by Lemma B.3. These demonstrate $\nu\tilde{c}.(P_0|P_1) \xrightarrow{\nu\tilde{c_1}.\overline{a}\langle V \rangle} \nu\tilde{c_3}.(P_0|P_1')$ and $\nu\tilde{d}.(Q_0|Q_1) \xrightarrow{\nu\tilde{d_1}.\overline{b}\langle W \rangle} \nu\tilde{d_3}.(Q_0|Q_1')$, where $\{\tilde{c_3}\} = \{\tilde{c}\} \setminus \{\tilde{c_1}\}$, $\{\tilde{d_2}\} = \{\tilde{d_1}\} \setminus \{\tilde{d}\}$ and $\{\tilde{d_3}\} = \{\tilde{d}\} \setminus \{\tilde{d_1}\}$. By clause 7, we have $P_0 \mathcal{Y}_{\mathcal{E}' \cup \{(\tilde{c_2}, \tilde{d_2})\}} Q_0$. By $\tilde{c_3} \notin \mathsf{fn}(V)$ and $\tilde{d_3} \notin \mathsf{fn}(W)$, we have $\mathcal{E} \cup \widehat{\{(V,W)\}} \subseteq \{(V,W) \mid V(\mathcal{E}' \cup \widehat{\{(\tilde{c_2}, \tilde{d_2})\}})^* W \, and \mathsf{fn}(V) \cap \{\tilde{c_3}\} = \mathsf{fn}(W) \cap \{\tilde{d_3}\} = \emptyset\}$. Therefore $\nu\tilde{c_3}.(P_0|P_1') \mathcal{Y}^{(*)}_{\mathcal{E} \cup \{(V,W)\}} \nu\tilde{d_3}.(Q_0|Q_1')$, i.e.,
$\nu\tilde{c_3}.(P_0|P_1') \mathcal{X}_{\mathcal{E} \cup \{(V,W)\}} \nu\tilde{d_3}.(Q_0|Q_1')$.

Case 4: Similar to clause (1–3).

Case 5: $V_1 \hat{\mathcal{E}} W_1 \quad V_2 \hat{\mathcal{E}} W_2$

We have $V_1 \hat{\mathcal{E}'}^* W_1$ and $V_2 \hat{\mathcal{E}'}^* W_2$. The difference between V_1 and W_1 or V_2 and W_2 is that of $\hat{\mathcal{E}'}$. Assume $V_1 = C_1[\tilde{V_1}]$, $V_2 = C_2[\tilde{V_2}]$, $W_1 = C_1[\tilde{W_1}]$, $W_2 = C_2[\tilde{W_2}]$, $\tilde{V_1} \hat{\mathcal{E}'} \tilde{W_1}$, and $\tilde{V_2} \hat{\mathcal{E}'} \tilde{W_2}$. Then by clause 5 of $\mathcal{Y}$, we have $V_1 = V_2 \iff \tilde{V_1} = \tilde{V_2} \iff \tilde{W_1} = \tilde{W_2} \iff W_1 = W_2$. Hence $V_1 = V_2 \iff W_1 = W_2$.

Case 6: ‘$P'\hat{\mathcal{E}}$‘Q'
By $\hat{\mathcal{E}} \subseteq \hat{\mathcal{E}'}^*$, we have two subcases of the relation between ‘P' and ‘Q'.
Subcase: ‘$P'\hat{\mathcal{E}'}$‘Q'
By clause 6 of $\mathcal{Y}$, we have $P_0|P'\mathcal{Y}^{(*)}_{\mathcal{E}'}Q_0|Q'$. Then we have $P|P' \equiv \nu\tilde{c}.(P_0|P'|P_1)\mathcal{Y}^{(*)}_{\mathcal{E}'}\nu\tilde{d}.(Q_0|Q'|Q_1) \equiv Q|Q'$, hence $P|P'\mathcal{Y}^{(*)}_{\mathcal{E}'}Q|Q'$, i.e., $P|P'\mathcal{X}_{\mathcal{E}}Q|Q'$.
Subcase: otherwise
Assume that ‘$P'\hat{\mathcal{E}'}^*$‘Q'holds. Then we have $P'\hat{\mathcal{E}'}^*Q'$. By $\tilde{c} \notin \mathsf{fn}(P')$, $\tilde{d} \notin \mathsf{fn}(Q')$ and $P_0\mathcal{Y}_{\mathcal{E}'}Q_0$, we have $P|P' \equiv \nu\tilde{c}.(P_0|P_1|P')\mathcal{Y}^{(*)}_{\mathcal{E}}\nu\tilde{d}.(Q_0|Q_1|Q') \equiv Q|Q'$. Therefore we have $P|P'\mathcal{X}_{\mathcal{E}}Q|Q'$.

Case 7: $a \notin \mathsf{fn}(P, \#_1(\mathcal{E})) \quad b \notin \mathsf{fn}(Q, \#_2(\mathcal{E}))$
We can assume that $a \notin \mathsf{fn}(P_0, \#_1(\mathcal{E}'))$ and $b \notin \mathsf{fn}(Q_0, \#_2(\mathcal{E}'))$ hold. By clause 7 of $\mathcal{Y}$, it holds that $P_0\mathcal{Y}_{\mathcal{E}'\cup\{(a,b)\}}Q_0$. By $\hat{\mathcal{E}'} \subseteq \widehat{\mathcal{E}' \cup \{(a,b)\}}$, we have $P_1\widehat{\mathcal{E}' \cup \{(a,b)\}}Q_1$. By $\mathcal{E} \cup \widehat{\{(a,b)\}} \subseteq \{(V,W) \mid V(\widehat{\mathcal{E}' \cup \{(a,b)\}})^*W\, and \tilde{c} \notin \mathsf{fn}(V), \tilde{d} \notin \mathsf{fn}(W)\}$, we obtain $P\mathcal{X}_{\mathcal{E}\cup\{(a,b)\}}Q$.

Case 8: $V\hat{\mathcal{E}}W$
Subcase 8a, 8b, 8c: $V = a, f, \hat{f}(V_1, \ldots, V_l)$
If $V\hat{\mathcal{E}'}W$, then they hold immediately by clause 8a, 8b, 8c, respectively. If $V\hat{\mathcal{E}'}^*W$ and not $V\hat{\mathcal{E}'}W$, the outermost syntax of V and W are same, since these are of the context. Therefore the conditions hold.
Subcase 8d: $V \in$ **Quo**
We can assume that $b \notin \mathsf{fn}(\mathcal{E}')$ holds. Then we have $\exists b \notin \mathsf{fn}(\mathcal{E}').P_0\mathcal{Y}^{(*)}_{\mathcal{E}'\cup\{(reify_b(V), reify_b(W))\}}Q_0$ by clause 8d of $\mathcal{Y}$ and $P_0\mathcal{Y}_{\mathcal{E}'}Q_0$, since $b \notin \mathsf{fn}(\mathcal{E}')$. By $\mathsf{fn}(V) \subseteq \mathsf{fn}(\#_1(\mathcal{E}))$ and $\mathsf{fn}(W) \subseteq \mathsf{fn}(\#_2(\mathcal{E}))$, we have $\mathcal{E} \cup \widehat{\{(reify_b(V), reify_b(W))\}} \subseteq \{(V', W') \mid V'(\widehat{\mathcal{E}' \cup \{(reify_b(V), reify_b(W))\}})^*W' and \mathsf{fn}(V') \cap \{\tilde{c}\} = \mathsf{fn}(W') \cap \{\tilde{d}\} = \emptyset\}$. Therefore $\exists b \notin \mathsf{fn}(\mathcal{E}).P\mathcal{Y}^{(*)}_{\mathcal{E}\cup\{(reify_b(V), reify_b(W))\}}Q$, i.e., $P\mathcal{X}_{\mathcal{E}\cup\{(reify_b(V), reify_b(W))\}}Q$.

Case 9: Similar to clause 8. □

B.3 Soundness of the system

Definition B.1 (reduction-closed barbed equivalence). *Reduction-closed barbed equivalence is the largest binary relation $\approx$ on closed processes such that $P \approx Q$ implies:*

1. $P \xrightarrow{\tau} P'$ *implies* $Q \xrightarrow{\tau} \ldots \xrightarrow{\tau} Q'$ *and* $P' \approx Q'$
2. $P \downarrow_\mu$ *implies* $Q \Downarrow_\mu$
3. *the converse of 1 and 2 on* Q
4. $P|R \approx Q|R$ *for all processes* R

Theorem B.2 (soundness of environmental bisimulation). *If $P \simeq Q$, then $P \approx Q$.*

Proof. We have to prove 4 cases of the requirement of reduction-closed barbed equivalence. Suppose $P \simeq Q$. Then we can suppose that there exists $\mathcal{X}$ such that $P\mathcal{X}_{\mathcal{E}}Q$, $\mathcal{E} = \{(a,a) \mid a \in \mathsf{fn}(P,Q)\}$ and $\mathcal{X}$

is an environmental bisimulation.

Case 1: $P \xrightarrow{\tau} P'$
By clause 1 of environmental bisimulation, we have $\exists Q'.Q \xrightarrow{\tau} \dots \xrightarrow{\tau} Q'$ and $P'\mathcal{X}_{\mathcal{E}}Q'$. Take $\mathcal{Y} = \{(\mathcal{E}', P, Q) \mid \mathcal{E}' \subseteq \mathcal{E}, P\mathcal{X}_{\mathcal{E}}Q\}$. Then $\mathcal{Y}$ is also environmental bisimulation. Therefore $P'\mathcal{Y}_{\mathcal{E}'}Q'$ and $\mathcal{E}' = \{(a,a) \mid a \in \mathsf{fn}(P', Q')\} \subseteq \mathcal{E}$. Hence $P' \simeq Q'$.

Case 2: $P \downarrow_\mu$
There are 2 subcases of μ.
Subcase: $P \downarrow_a$
We have $P \xrightarrow{a(V)} P'$ for some V, P'. It indicates that for some $V\hat{\mathcal{E}}^*W$, we have $P \xrightarrow{a(V)} P'$. By clause 2 of environmental bisimulation, we have $Q \xrightarrow{\tau} \dots \xrightarrow{a(W)} \dots \xrightarrow{\tau} Q'$ for some Q', since $a \in \mathsf{fn}(P,Q)$ and $a\hat{\mathcal{E}}a$. Therefore $Q \Downarrow_a$.
Subcase: $P \downarrow_{\overline{a}}$
We have $P \xrightarrow{\nu\tilde{c}.\overline{a}\langle V\rangle} P'$ for some $\tilde{c}, V, P'$. We can choose $\tilde{c}$ such that $\tilde{c} \notin \mathsf{fn}(\#_1(\mathcal{E}))$. So by clause 3 of environmental bisimulation, we have $Q \xrightarrow{\tau} \dots \xrightarrow{\nu\tilde{d}.\overline{a}\langle W\rangle} \dots \xrightarrow{\tau} Q'$ for some $W, Q', \tilde{d}$, since $a\hat{\mathcal{E}}a$. Therefore $Q \Downarrow_a$.

Case 3: Similar to 1 and 2.

Case 4: R is a process
Suppose $P \simeq Q$, i.e., $P \sim_{\mathcal{E}} Q$ for $\mathcal{E} = \{(a,a) \mid a \in \mathsf{fn}(P,Q)\}$. Let $\mathcal{E}' = \{(b,b) \mid b \in \mathsf{fn}(R)\}$. By clause 7 of environmental bisimulation, $P \sim_{\mathcal{E}\cup\mathcal{E}'} Q$. Since $R(\mathcal{E}\cup\mathcal{E}')^*R$, we have $P|R \sim^{(*)}_{\mathcal{E}\cup\mathcal{E}'} Q|R$ by Definition A.2. Since $\sim$ is an environmental bisimulation up-to context, $P|R \sim_{\mathcal{E}\cup\mathcal{E}'} Q|R$ by Theorem B.1. Hence $P|R \simeq Q|R$. ☐

B.4 Completeness of the system

Lemma B.5.

$$\begin{aligned}\mathcal{X} = \{(\mathcal{E}, P, Q) \quad | \quad & \forall\tilde{a} \notin \mathsf{fn}(\mathcal{E}, P, Q).\\ & \nu\tilde{c}.(P|!\overline{a_1}\langle V_1\rangle|\dots|!\overline{a_n}\langle V_n\rangle) \approx \nu\tilde{d}.(Q|!\overline{a_1}\langle W_1\rangle|\dots|!\overline{a_n}\langle W_n\rangle),\\ & \{\tilde{c}\} = \mathsf{fn}(P,\tilde{V}), \{\tilde{d}\} = \mathsf{fn}(Q,\tilde{W}), \mathcal{E} = \{(\tilde{V},\tilde{W})\},\\ & \exists\tilde{b} \notin \mathcal{E}.(\mathcal{E}\cup\{\widehat{(\mathit{reify}_{\tilde{b}}(\tilde{V}), \mathit{reify}_{\tilde{b}}(\tilde{W}))}\} \subseteq \mathcal{E}\cup\widehat{\{(\tilde{b},\tilde{b})\}})\}\end{aligned}$$

is an environmental bisimulation up-to context.

Proof. Let $\tilde{a} \notin \mathsf{fn}(\mathcal{E}, P, Q), \nu\tilde{c}.(P|!\overline{a_1}\langle V_1\rangle|\dots|!\overline{a_n}\langle V_n\rangle) \approx \nu\tilde{d}.(Q|!\overline{a_1}\langle W_1\rangle|\dots|!\overline{a_n}\langle W_n\rangle), \{\tilde{c}\} = \mathsf{fn}(P,\tilde{V})$, $\{\tilde{d}\} = \mathsf{fn}(Q,\tilde{W})$, $\tilde{V}\mathcal{E}\tilde{W}$, and $\exists\tilde{b} \notin \mathcal{E}.(\mathcal{E}\cup\{\widehat{(\mathit{reify}_{\tilde{b}}(\tilde{V}), \mathit{reify}_{\tilde{b}}(\tilde{V}))}\} \subseteq \mathcal{E}\cup\widehat{\{(\tilde{b},\tilde{b})\}})$. We check the conditions of environmental bisimulation up-to context.

Case 1: $P \xrightarrow{\tau} P'$
By $\nu\tilde{c}.(P|!\overline{a_1}\langle V_1\rangle|\dots|!\overline{a_n}\langle V_n\rangle) \approx \nu\tilde{d}.(Q|!\overline{a_1}\langle W_1\rangle|\dots|!\overline{a_n}\langle W_n\rangle)$, there exists Q' such that $Q \xrightarrow{\tau} Q'$ and

$\nu\tilde{c}.(P'|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle) \approx \nu\tilde{d}.(Q'|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle)$. Thus $P'\mathcal{X}_{\mathcal{E}}Q'$, i.e., $P'\mathcal{X}_{\mathcal{E}}^{(*)}Q'$.

Case 2: $P \xrightarrow{a(V)} P' \quad a\hat{\mathcal{E}}b \quad V\hat{\mathcal{E}}^*W$
Assume that $R = a_i(x_i).\ldots.a_j(x_j).\overline{M}\langle N\rangle.(m(x)|\overline{m}\langle\_\rangle)$, where m is fresh (i.e., $m \notin \mathsf{fn}(P, Q, \mathcal{E}, \tilde{a})$), $eval(\{^{V_i,\ldots,V_j}/_{x_i,\ldots,x_j}\}M) = a$, $eval(\{^{W_i,\ldots,W_j}/_{x_i,\ldots,x_j}\}M) = b$, $eval(\{^{V_i,\ldots,V_j}/_{x_i,\ldots,x_j}\}N) = V$, and $eval(\{^{W_i,\ldots,W_j}/_{x_i,\ldots,x_j}\}N) = W$. Then we have $\nu\tilde{c}.(P|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle)|R \xrightarrow{\tau} \ldots \xrightarrow{\tau}\equiv$
$\nu\tilde{c}.(P|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle)|\overline{\{^{V_i,\ldots,V_j}/_{x_i,\ldots,x_j}\}M}\langle\{^{V_i,\ldots,V_j}/_{x_i,\ldots,x_j}\}N\rangle.(d(x)|\overline{d}\langle\_\rangle) \xrightarrow{\tau}$
$\nu\tilde{c}.(P'|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle|d(x)|\overline{d}\langle\_\rangle) \xrightarrow{\tau}\equiv \nu\tilde{c}.(P'|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle)$.
By $\nu\tilde{c}.(P|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle) \approx \nu\tilde{d}.(Q|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle)$, there exists Q' such that
$\nu\tilde{d}.(Q|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle)|R \xrightarrow{\tau} \ldots \xrightarrow{\tau}\equiv$
$\nu\tilde{d}.(Q|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle)|\overline{\{^{W_i,\ldots,W_j}/_{x_i,\ldots,x_j}\}M}\langle\{^{W_i,\ldots,W_j}/_{x_i,\ldots,x_j}\}N\rangle.(d(x)|\overline{d}\langle\_\rangle) \xrightarrow{\tau}\equiv$
$\nu\tilde{d}.(Q'|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle)$ and $\nu\tilde{c}.(P'|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle) \approx \nu\tilde{d}.(Q'|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle)$. Thus $Q \xrightarrow{\tau} \ldots \xrightarrow{b(W)} \ldots \xrightarrow{\tau} Q'$ and $P'\mathcal{X}_{\mathcal{E}}^{(*)}Q'$.

Case 3: $P \xrightarrow{\nu\tilde{c}_1.\overline{a}\langle V\rangle} P' \quad a\hat{\mathcal{E}}b \quad \tilde{c}_1 \notin \mathsf{fn}(\#_1(\mathcal{E}))$
Similar to Case 2. We can take R and $\mathcal{E}'$ such that $R = a_i(x_i).\ldots.a_j(x_j).M(x).match\ M_1\ as\ y_1\ in\ \ldots$ $match\ M_l\ as\ y_l\ in\ ((\overline{m_1}\langle N_1\rangle|m_1(z_1).!\overline{a_{n+1}}\langle z_1\rangle)|\ldots|(\overline{m_k}\langle N_k\rangle|m_k(z_k).!\overline{a_{n+k}}\langle z_k\rangle))$ where $\tilde{m}$ is fresh, $eval(\{^{V_i,\ldots,V_j}/_{x_i,\ldots,x_j}\}M) = a$, $eval(\{^{W_i,\ldots,W_j}/_{x_i,\ldots,x_j}\}M) = b$,
$eval(\{^{V_i,\ldots,V_j}/_{x_i,\ldots,x_j}\}\{^{reify_{e_1}(M_1),\ldots reify_{e_l}(M_l)}/_{y_1,\ldots,y_l}\}\tilde{N}) = \tilde{V}'$,
$eval(\{^{W_i,\ldots,W_j}/_{x_i,\ldots,x_j}\}\{^{reify_{e_1}(M_1),\ldots reify_{e_l}(M_l)}/_{y_1,\ldots,y_l}\}\tilde{N}) = \tilde{W}'$, $\tilde{V}'\mathcal{E}'\tilde{W}'$, $\mathcal{E} \cup \widehat{\{(V,W)\}} \subseteq \widehat{\mathcal{E}'}$, and $\forall V''\mathcal{E}'W''.(\mathcal{E}' \cup \{\widehat{(reify_{\tilde{e}}(\tilde{V}''), reify_{\tilde{e}}(\tilde{W}''))}\} \subseteq \widehat{\mathcal{E}'})$. Then we have
$\nu\tilde{c}.(P|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle)|R \xrightarrow{\tau} \ldots \xrightarrow{\tau}\equiv$
$\nu\tilde{c'}.(P|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle|((\overline{m_1}\langle\{^{V_i,\ldots,V_j}/_{x_i,\ldots,x_j}\}\{^{reify_{e_1}(M_1),\ldots reify_{e_l}(M_l)}/_{y_1,\ldots,y_l}\}N_1\rangle|$
$m_1(z_1).!\overline{a_{n+1}}\langle z_1\rangle)|\ldots|(\overline{m_k}\langle\{^{V_i,\ldots,V_j}/_{x_i,\ldots,x_j}\}\{^{reify_{e_1}(M_1),\ldots reify_{e_l}(M_l)}/_{y_1,\ldots,y_l}\}N_k\rangle|m_k(z_k).!\overline{a_{n+k}}\langle z_k\rangle))) \xrightarrow{\tau}$
$\ldots \xrightarrow{\tau}\equiv \nu\tilde{c'}.(P|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle|!\overline{a_{n+1}}\langle V_1'\rangle|\ldots|!\overline{a_{n+k}}\langle V_k'\rangle)$.
By $\nu\tilde{c}.(P|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle) \approx \nu\tilde{d}.(Q|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle)$, there exists Q' such that
$\nu\tilde{d}.(Q|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle)|R \xrightarrow{\tau} \ldots \xrightarrow{\tau}\equiv \nu\tilde{d'}.(Q'|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle|!\overline{a_{n+1}}\langle W_1'\rangle|\ldots|!\overline{a_{n+k}}\langle W_k'\rangle)$ and $\nu\tilde{c'}.(P|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle|!\overline{a_{n+1}}\langle V_1'\rangle|\ldots|!\overline{a_{n+k}}\langle V_k'\rangle) \approx$
$\nu\tilde{d'}.(Q'|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle|!\overline{a_{n+1}}\langle W_1'\rangle|\ldots|!\overline{a_{n+k}}\langle W_k'\rangle)$. Thus $Q \xrightarrow{\tau} \ldots \xrightarrow{\nu\tilde{d'}-\tilde{d}.\overline{b}\langle W\rangle} \ldots \xrightarrow{\tau} Q'$ and $P'\mathcal{X}_{\mathcal{E}'}Q'$, i.e., $P'\mathcal{X}_{\mathcal{E}\cup\{(V,W)\}}^{(*)}Q'$.

Case 4: Similar to clause (1–3).

Case 5: $V_1\hat{\mathcal{E}}W_1 \quad V_2\hat{\mathcal{E}}W_2$
Assume that $R = a_i(x_i).\ldots.a_j(x_j).if\ M_1 = M_2\ then\ b(x)\ else\ c(y)$ where b and c are fresh, $b \neq c$, $eval(\{^{V_i,\ldots,V_j}/_{x_i,\ldots,x_j}\}M_1) = V_1$, $eval(\{^{W_i,\ldots,W_j}/_{x_i,\ldots,x_j}\}M_1) = W_1$, $eval(\{^{V_i,\ldots,V_j}/_{x_i,\ldots,x_j}\}M_2) = V_2$, and $eval(\{^{W_i,\ldots,W_j}/_{x_i,\ldots,x_j}\}M_2) = W_2$. If $V_1 = V_2$, then $\nu\tilde{c}.(P|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle)|R \Downarrow_b$, and $\not\Downarrow_c$. Thus we must have $W_1 = W_2$, so that $\nu\tilde{d}.(Q|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle)|R \Downarrow_b$, and $\not\Downarrow_c$. The other cases are similar.

Case 6: $`(P')\hat{\mathcal{E}}`(Q')$
By the assumption of $\mathcal{E}$, we have $P'\hat{\mathcal{E}}^*Q'$. Therefore $P|P'\mathcal{X}_{\mathcal{E}}^{(*)}Q|Q'$.

Case 7: $a \notin \mathsf{fn}(P, \#_1(\mathcal{E})) \quad b \notin \mathsf{fn}(Q, \#_2(\mathcal{E}))$
Assume that $R = \nu a.!\overline{a_{n+1}}\langle a\rangle$. Then we have $\nu\tilde{c}.(P|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle)|R \equiv \nu\tilde{c}, a.(P|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle|!\overline{a_{n+1}}\langle a\rangle)$ and $\nu\tilde{d}.(Q|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle)|R \equiv \nu\tilde{c}, b.(Q|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle|!\overline{a_{n+1}}\langle b\rangle)$. Therefore we have $P\mathcal{X}_{\mathcal{E}\cup\{(a,b)\}}Q$.

Case 8: $V\hat{\mathcal{E}}W$
Subcase 8a, 8b, 8c: $V = a, f, \hat{f}(V_1, \ldots, V_l)$
Assume that $R = a_i(x_i).\ldots.a_j(x_j).match\, M\, as\, x\, in\, if\, \#_1(x) = S\, then\, c(x)\, else\, d(y)$, where $S =$ `name`, `fun`, or `capp`, $eval(\{^{V_i,\ldots,V_j}/_{x_i,\ldots,x_j}\}M) = V$, and $eval(\{^{W_i,\ldots,W_j}/_{x_i,\ldots,x_j}\}M) = W$. If $V = a$, then we have $\nu\tilde{c}.(P|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle)|R \Downarrow_c$ and $\not\Downarrow_d$. Thus we have $W = b$, so that $\nu\tilde{d}.(Q|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle)|R \Downarrow_c$, and $\not\Downarrow_d$. The other cases are similar.
Subcase 8d: $V \in$ **Quo**
By clause 7 of environmental bisimulation up-to context, we have some b such that $P\mathcal{X}_{\mathcal{E}\cup\{(b,b)\}}Q$, so that $\mathcal{E} \cup \{(reify_b\widehat{(V), reify_b}(W))\} \subseteq \mathcal{E} \cup \widehat{\{(b,b)}\}$. Therefore we have $P\mathcal{X}^{(*)}_{\mathcal{E}\cup\{(reify_b(V), reify_b(W))\}}Q$.

Case 9: Similar to clause 8. $\square$

Lemma B.6. $\mathcal{R} = \{(P, Q) \mid P \equiv \nu\tilde{b}.P_1, Q \equiv \nu\tilde{b}.Q_1, P_1 \approx Q_1\} \subseteq \approx$.

Proof. Assume $P \equiv \nu\tilde{b}.P_1$, $Q \equiv \nu\tilde{b}.Q_1$, and $P_1 \approx Q_1$. We check the conditions of reduction-closed barbed equivalence.

Case 1: $P \xrightarrow{\tau} P'$
By Lemma B.1, we have $\nu\tilde{b}.P_1 \xrightarrow{\tau} \nu\tilde{b}.P_1'$, where $P_1 \xrightarrow{\tau} P_1'$. Then there exists Q_1' such that $Q_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} Q_1'$ and $P_1' \approx Q_1'$ by clause 1, thus we have $\nu\tilde{b}.Q_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} \nu\tilde{b}.Q_1'$. If we assume $P' \equiv \nu\tilde{b}.P_1'$ and $Q \xrightarrow{\tau} \ldots \xrightarrow{\tau} Q' \equiv \nu\tilde{b}.Q_1'$, we have $P'\mathcal{R}Q'$, since $P_1' \approx Q_1'$.

Case 2: $P \downarrow_\mu$
We have $P_1 \downarrow_\mu$ by $P \downarrow_\mu$, where $\tilde{b} \notin \mathsf{fn}(\mu)$. Then by $P_1 \approx Q_1$, we have $Q_1 \Downarrow_\mu$. By $\tilde{b} \notin \mathsf{fn}(\mu)$, we have $Q \Downarrow_\mu$.

Case 3: Similar to 1 and 2.

Case 4: R is a process
There exist $\tilde{b'}$ such that $\nu\tilde{b}.P_1 \equiv \nu\tilde{b'}.\{^{b'}/_b\}P_1$, $\nu\tilde{b}.Q_1 \equiv \nu\tilde{b'}.\{^{b'}/_b\}Q_1$, and $\{^{b'}/_b\}P_1 \approx \{^{b'}/_b\}Q_1$, where $\tilde{b'} \notin \mathsf{fn}(R)$. Then we have $\nu\tilde{b}.P_1|R \equiv \nu\tilde{b'}.(\{^{b'}/_b\}P_1|R)$ and $\nu\tilde{b}.Q_1|R \equiv \nu\tilde{b'}.(\{^{b'}/_b\}Q_1|R)$. By $\{^{b'}/_b\}P_1 \approx \{^{b'}/_b\}Q_1$ and clause 4, we have $\{^{b'}/_b\}P_1|R \approx \{^{b'}/_b\}Q_1|R$. Therefore we have $P|R \equiv \nu\tilde{b'}.(\{^{b'}/_b\}P_1|R)\mathcal{R}\nu\tilde{b'}.(\{^{b'}/_b\}Q_1|R) \equiv Q|R$. Hence $P|R\mathcal{R}Q|R$.
$\square$

Corollary B.1. *If $P \approx Q$, then $\nu\tilde{b}.P \approx \nu\tilde{b}.Q$.*

Theorem B.3 (completeness of environmental bisimulation). *If $P \approx Q$, then $P \simeq Q$.*

Proof. Assume that

$$\begin{aligned}\mathcal{X} = \{(\mathcal{E}, P, Q) \quad | \quad & \forall \tilde{a} \notin \mathsf{fn}(\mathcal{E}, P, Q). \\ & \nu\tilde{c}.(P|!\overline{a_1}\langle V_1\rangle|\ldots|!\overline{a_n}\langle V_n\rangle) \approx \nu\tilde{d}.(Q|!\overline{a_1}\langle W_1\rangle|\ldots|!\overline{a_n}\langle W_n\rangle), \\ & \{\tilde{c}\} = \mathsf{fn}(P, \tilde{V}), \{\tilde{d}\} = \mathsf{fn}(Q, \tilde{W}), \mathcal{E} = \{(\tilde{V}, \tilde{W})\}, \\ & \exists\tilde{b} \notin \mathcal{E}.(\mathcal{E} \cup \widehat{\{(reify_{\tilde{b}}(\tilde{V}), reify_{\tilde{b}}(\tilde{W}))\}} \subseteq \mathcal{E} \cup \widehat{\{(\tilde{b}, \tilde{b})\}})\}\end{aligned}$$

Then we have $\mathcal{X}$ is an environmental bisimulation up-to context by Lemma B.5. By the way, if $P \approx Q$, we have $P|!\overline{a_1}\langle b_1\rangle|\ldots|!\overline{a_n}\langle b_n\rangle \approx Q|!\overline{a_1}\langle b_1\rangle|\ldots|!\overline{a_n}\langle b_n\rangle$ by clause 4 of reduction-closed barbed equivalence, and $\nu\tilde{b}.(P|!\overline{a_1}\langle b_1\rangle|\ldots|!\overline{a_n}\langle b_n\rangle) \approx \nu\tilde{b}.(Q|!\overline{a_1}\langle b_1\rangle|\ldots|!\overline{a_n}\langle b_n\rangle)$ by Corollary B.1, where $\{\tilde{b}\} = \mathsf{fn}(P, Q)$ and $\tilde{a}$ are fresh. Therefore $P\mathcal{X}_{\mathcal{E}}Q$ where $\mathcal{E} = \{(b, b) \mid b \in \mathsf{fn}(P, Q)\}$, that is, $P \simeq Q$. ☐