Foundations of Software Science (ソフトウェア基礎科学) /
Foundations of Computer Software (ソフトウェア基礎)
Reference Handout
November 13, 2009
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Simply typed λ-calculus

The syntax of λ-terms M, N, ... is given by:

M	::=	i	(integers)
	|	x	(variables)
	|	$\lambda x.M$	(functions)
	|	$M_1\ M_2$	(function applications)
	|	(M_1,M_2)	(pairs)
	|	fst(M)	(first projections)
	|	snd(M)	(second projections)
	|	Left(M)	(left variants)
	|	Right(M)	(right variants)
	|	case M of Left(x)$\Rightarrow N_1$ | Right(y)$\Rightarrow N_2$	(case branches)

The syntax of types τ, σ, ... is:

τ	::=	int	(integer type)
	|	$\tau_1 \rightarrow \tau_2$	(function types)
	|	$\tau_1 \times \tau_2$	(pair types)
	|	$\tau_1 + \tau_2$	(variant types)

A type environment Γ, Δ, ... is a (partial) map from variables to types. It is often written like

$$\Gamma = x_1{:}\tau_1,\ x_2{:}\tau_2,\ \ldots,\ x_n{:}\tau_n$$

when the domain of Γ is $\{ x_1,x_2,\ldots,x_n \}$ and $\Gamma(x_i) = \tau_i$ for $i = 1,2,\ldots,n$.

A type judgment $\Gamma \vdash M : \tau$ is the smallest relation among type environments, λ-terms and types that satisfies the typing rules in page 3.

Intuitionistic propositional logic (without negations)

The syntax of propositions P, Q, R, ... is:

$P ::=$	A	(atomic formulas)
$\|$	$P \Rightarrow Q$	(implications)
$\|$	$P \wedge Q$	(conjunctions)
$\|$	$P \vee Q$	(disjunctions)

An assumption Γ is a set of propositions.

A judgment $\Gamma \vdash P$ is the smallest relation between assumptions and propositions that satisfies the natural deduction rules in page 4.

Suggested reference (besides the blackboard in class): Sørensen and Urzyczyn, Lectures on the Curry-Howard isomorphism, ISBN 978-0-44452-077-7, Section 2.2 (Natural deduction), 3.1 (Simply typed λ-calculus a la Curry) and 4.5 (Sums and products). Draft available online: http://folli.loria.fr/cds/1999/library/pdf/curry-howard.pdf

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}\ \text{T-Var}$$

$$\frac{\Gamma, x{:}\tau_1 \vdash M : \tau_2}{\Gamma \vdash \lambda x.M : \tau_1 \rightarrow \tau_2}\ \text{T-Fun}$$

$$\frac{\Gamma \vdash M_1 : \tau_1 \rightarrow \tau_2 \qquad \Gamma \vdash M_2 : \tau_1}{\Gamma \vdash M_1\ M_2 : \tau_2}\ \text{T-App}$$

$$\frac{\Gamma \vdash M_1 : \tau_1 \qquad \Gamma \vdash M_2 : \tau_2}{\Gamma \vdash (M_1, M_2) : \tau_1 \times \tau_2}\ \text{T-Pair}$$

$$\frac{\Gamma \vdash M : \tau_1 \times \tau_2}{\Gamma \vdash \mathrm{fst}(M) : \tau_1}\ \text{T-Fst}$$

$$\frac{\Gamma \vdash M : \tau_1 \times \tau_2}{\Gamma \vdash \mathrm{snd}(M) : \tau_2}\ \text{T-Snd}$$

$$\frac{\Gamma \vdash M : \tau_1}{\Gamma \vdash \mathrm{Left}(M) : \tau_1 + \tau_2}\ \text{T-Left}$$

$$\frac{\Gamma \vdash M : \tau_2}{\Gamma \vdash \mathrm{Right}(M) : \tau_1 + \tau_2}\ \text{T-Right}$$

$$\frac{\Gamma \vdash M : \tau_1 + \tau_2 \qquad \Gamma, x{:}\tau_1 \vdash N_1 : \tau \qquad \Gamma, y{:}\tau_2 \vdash N_2 : \tau}{\Gamma \vdash (\mathrm{case}\ M\ \mathrm{of}\ \mathrm{Left}(x) \Rightarrow N_1 \mid \mathrm{Right}(y) \Rightarrow N_2) : \tau}\ \text{T-Case}$$

$$\frac{}{\Gamma \vdash i : \mathrm{int}}\ \text{T-Int}$$

$$\frac{P \in \Gamma}{\Gamma \vdash P}\ \text{Axiom}$$

$$\frac{\Gamma, P \vdash Q}{\Gamma \vdash P \Rightarrow Q}\ \Rightarrow\text{-Intro}$$

$$\frac{\Gamma \vdash P \Rightarrow Q \qquad \Gamma \vdash P}{\Gamma \vdash Q}\ \Rightarrow\text{-Elim}$$

$$\frac{\Gamma \vdash P \qquad \Gamma \vdash Q}{\Gamma \vdash P \wedge Q}\ \wedge\text{-Intro}$$

$$\frac{\Gamma \vdash P \wedge Q}{\Gamma \vdash P}\ \wedge\text{-Elim-L}$$

$$\frac{\Gamma \vdash P \wedge Q}{\Gamma \vdash Q}\ \wedge\text{-Elim-R}$$

$$\frac{\Gamma \vdash P}{\Gamma \vdash P \vee Q}\ \vee\text{-Intro-L}$$

$$\frac{\Gamma \vdash Q}{\Gamma \vdash P \vee Q}\ \vee\text{-Intro-R}$$

$$\frac{\Gamma \vdash P \vee Q \qquad \Gamma, P \vdash R \qquad \Gamma, Q \vdash R}{\Gamma \vdash R}\ \vee\text{-Elim}$$