

Foundations of Software Science (ソフトウェア基礎科学) /  
Foundations of Computer Software (ソフトウェア基礎)

Second Preliminary Quiz (Anonymous)

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Eijiro Sumii (Instructor)

Answer "(always) true", "(always) false", or "depends (on P, Q)" for each of the following logical formulas (in classical propositional or predicate logic).

1.  $P \vee \neg P$  True
2.  $P \wedge \neg P$  False
3.  $P \wedge Q \Rightarrow P \vee Q$  True
4.  $P \vee Q \Rightarrow P$  Depends
5.  $P(x) \wedge \forall y. \neg P(y)$  False
6.  $P(x) \vee \exists y. P(y)$  Depends
7.  $(\exists x. P(x)) \wedge (\exists x. Q(x)) \Rightarrow (\exists x. (P(x) \wedge Q(x)))$  Depends
8.  $(\forall x. (P(x) \vee Q(x))) \Rightarrow (\forall x. P(x)) \vee (\forall x. Q(x))$  Depends
9.  $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$  True
10.  $(\forall x. (P \Rightarrow Q(x))) \Rightarrow (P \Rightarrow (\forall x. Q(x)))$  True  
(Assume that  $x$  does not appear in  $P$ .)

(以上の問題は東京大学理学部情報科学科「情報論理」各年度の中間試験および追試問題の間1より抜粋して簡単にしたものです。)

Write down the following sets (only) by using the set comprehension notation, predicate logic operators, and arithmetic expressions. Use  $\mathbb{N}$  for the set of all natural numbers.

11. The set of all odd numbers.  $\{ x \in \mathbb{N} \mid \exists y \in \mathbb{N}. x=2y+1 \}$

12. The set of all prime numbers.  
 $\{ x \in \mathbb{N} \mid \neg(x=1) \wedge \forall y \in \mathbb{N}. y=1 \vee y=x \vee \neg \exists z \in \mathbb{N}. x=yz \}$