Theories of Information Hiding in Lambda-Calculus

Logical Relations and Bisimulations for Encryption and Type Abstraction

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Main Results

- Proof methods for two forms of information hiding in computer programs
  - Logical relations for perfect encryption
  - Bisimulations for perfect encryption
  - Bisimulations for type abstraction
    - First solution to a problem of 20 years

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Background

- Information hiding (or abstraction) is crucial for building large systems
  - ...including computer software!
- Type abstraction is the primary method of information hiding in programming languages
  - The basis of objects, modules, components, etc.

A Classical Example

```
interface Complex =
  type t
  fun make : real × real → t
  fun mul : t × t → t
  fun re : t → real
```

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Cartesian Implementation

```
module Cartesian : Complex =
  type t = real × real
  fun make(x,y) = (x,y)
  fun mul((x1,y1),(x2,y2)) =
     (x1*x2-y1*y2 , x1*y2+x2*y1)
  fun re(x,y) = x
```

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Polar Implementation

```
module Polar : Complex =
  type t = real × real
  fun make(x,y) =
    (sqrt(x*x+y*y),atan2(y,x))
  fun mul((r1,θ1),(r2,θ2)) =
    (r1*r2,θ1+θ2)
  fun re(r,θ) = r*cos(θ)
```

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The Abstraction Property

Contextual equivalence:

$$\vdash \texttt{Cartesian} \equiv \texttt{Polar} : \texttt{Complex}$$

I.e., the different implementations give the same result under any well-typed context in the language

- In this study, "result" means only the final value (or possible divergence)
  - Time, energy, rounding errors, etc. are out of scope

How to Prove it?

Logical relations [Reynolds 83]:

Induction on the type of the interface (`Complex` in previous example)

- So far, so good.

The Problems

- Logical relations become complex for expressive languages (e.g. with recursion or concurrency)
  - ...yet attackers must be expressed in the language (as well as users)
- Type abstraction doesn't work against untyped users/attackers
  - ...but we cannot "type-check the Internet"

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Our Solutions

- Use bisimulations in place of logical relations
- Use encryption instead of type abstraction

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Outline

1. **Logical relations for type abstraction**
   [Reynolds 83] [Mitchell 91]
2. **Bisimulations for type abstraction**
   [Sumii-Pierce, POPL'05]
3. **Bisimulations for encryption**
   [Sumii-Pierce, POPL'04 & TCS]
   - **Cf. Logical relations for encryption**
     [Sumii-Pierce, CSFW'01 & JCS]

Logical Relations (without Type Abstraction)

Relations between programs, defined by induction on their types

- Constants are related iff they are equal
- Tuples are related iff their elements are related
- Functions are related iff they map related arguments to related results

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Examples

- `123` and `123` are related at type `Int`
- `λx.x+1+2` and `λx.x+3` are related at type `Int→Int`
- `λx.(123,x+1+2)` and `λx.(123,x+3)` are related at type `Int→(Int×Int)`

Logical Relations for Type Abstraction

In addition to the previous cases:

- Abstract data of type α are related iff they are related by $\varphi(\alpha)$
  - where φ is a relation environment mapping abstract types to the relation between their implementations

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Example

Take

φ(`Complex.t`) =

{(`(x,y)`,`(r,θ)`) | x = r cos θ, y = r sin θ}

Then

$\varphi \vdash$ `Cartesian` ~ `Polar` : `Complex`

That is, `Cartesian` and `Polar` are logically related at type `Complex` under φ

Soundness of Logical Relations

Logical relations imply contextual equivalence

- Corollary of the "fundamental property" (a.k.a. the "basic lemma")
  - Proved by induction on the typing of terms, with assumptions on their free variables

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Shortcoming of Logical Relations

Don't "scale" to more expressive languages

- Recursive functions (or while-loops) complicate the fundamental property
- Recursive types complicate the definition of logical relations

Extra work required in every use of logical relations (cannot be done once and for all in meta theory)

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Bisimulations (without Type Abstraction)

Applicative bisimulations [Abramsky 90]:

Relations between values that satisfy some conditions to exclude inequivalent values

- Bisimilar constants are equal
- Bisimilar tuples have bisimilar elements
- Bisimilar functions return bisimilar results when applied to the same argument

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Examples

- {(`123`, `123`)} is a bisimulation
- {(`4`, `4`), (`5`, `5`)} is a bisimulation
- {(`(4,5)`,`(4,5)`), (`4`, `4`), (`5`, `5`)} is a bisimulation
- {(`λx.x+1+2`, `λx.x+3`), (`i`, `j`) | `i` = k+1+2, `j` = k+3, k : `int`} is a bisimulation
- Union of bisimulations is a bisimulation

Shortcoming of Applicative Bisimulations

Don't extend to type abstraction

- `Cartesian.re` and `Polar.re` do not return bisimilar results (i.e. the same real number) when applied to the same argument
  - They expect different representations

Bisimulations for Type Abstraction: First Try

Bisimilar functions return bisimilar results when applied to bisimilar arguments

- **No condition for abstract data themselves, as long as the other conditions are satisfied**

THIS IS UNSOUND!

- **Because contexts (users or attackers) can combine bisimilar values to make more complex arguments**

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Counter-Example

- `(1,fst)` and `(2,fst)` are not contextually equivalent at type $\alpha \times (\alpha \times \alpha \to \texttt{int})$

But

$\{(\texttt{(1,fst)}, \texttt{(2,fst)}, \alpha \times (\alpha \times \alpha \to \texttt{int})),$
$(\texttt{1}, \texttt{2}, \alpha),$
$(\texttt{fst}, \texttt{fst}, \alpha \times \alpha \to \texttt{int})\}$

satisfies all the bisimulation conditions so far!

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Bisimulations for Type Abstraction: Second Try

Bisimilar functions return bisimilar results when applied to $C[v_1,\ldots,v_n]$ and $C[v_1',\ldots,v_n']$

- for any bisimilar values $v_1,\ldots,v_n$ and $v_1',\ldots,v_n'$, and
- for any value context C.

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Example

$$
\begin{aligned}
R = \{\ & (\texttt{Cartesian}, \texttt{Polar}, \texttt{Complex}),\\
& (\texttt{Cartesian.make}, \texttt{Polar.make},\\
& \ \texttt{real} \times \texttt{real} \rightarrow \texttt{Complex.t}),\\
& (\texttt{Cartesian.mul}, \texttt{Polar.mul},\\
& \ \texttt{Complex.t} \times \texttt{Complex.t} \rightarrow \texttt{Complex.t}),\\
& (\texttt{Cartesian.re}, \texttt{Polar.re},\\
& \ \texttt{Complex.t} \rightarrow \texttt{real}),\\
& (\texttt{(x,y)}, \texttt{(r,}\theta\texttt{)}, \texttt{Complex.t}),\\
& (\texttt{z}, \texttt{z}, \texttt{real}) \mid \texttt{x} = \texttt{r}\cos\theta,\ \texttt{y} = \texttt{r}\sin\theta\ \}
\end{aligned}
$$

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The Last Problem

The previous definition is sound, but completeness is unclear

- **Because the union of two bisimulations wouldn't always be a bisimulation**
  - **Counter-example: the union $R \cup R^{-1}$ of the previous R and its inverse R^{-1}**

  $\Rightarrow$ Standard co-inductive method wouldn't apply

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Our Solution

Consider sets of relations as bisimulations

- **Intuition: Each relation represents a "world"**
- **Another intuition: Each relation represents the knowledge of an attacker, which increases by time (but nevertheless stays in the bisimulation)**
- **Also gives a natural account for the generativity of existential types**

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Examples

For the previous R,

- { R } is a bisimulation
- { R^{-1} } is another bisimulation
- { R, R^{-1} } is also a bisimulation
- { $R \cup R^{-1}$ } is <u>not</u> a bisimulation

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Soundness and Completeness

- Contextual equivalence is also generalized as a set of relations
- Then, it coincides with bisimilarity (the largest bisimulation)
  - Everything is formalized and proved in λ-calculus with full universal, existential, and recursive types (first result in 20 years!)

Other Examples

- Object encodings
- ML-like functors
- Higher-order polymorphic functions (the "dual" of abstract types)

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Idea: Abstraction by Encryption

Reinvention of dynamic sealing [Morris 73]

- **Secret key is generated for each abstract type**
- **Abstract data are encrypted when exported out of a module**
- **...and decrypted when imported back**

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Cartesian Implementation of `Complex` with Encryption

```
module Cartesian =
  fun make(x,y) = encrypt_k(x,y)
  fun mul(c_1,c_2) =
   let (x_1,y_1) = decrypt_k(c_1) in
   let (x_2,y_2) = decrypt_k(c_2) in
   encrypt_k(x_1*x_2-y_1*y_2 , x_1*y_2+x_2*y_1)
  fun re(c) =
   let (x,y) = decrypt_k(c) in x
```

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Polar Implementation of Complex with Encryption

```
module Polar =
  fun make(x,y) =
   encrypt_k'(sqrt(x*x+y*y),atan2(y,x))
  fun mul(c_1,c_2) =
   let (r_1,θ_1) = decrypt_k'(c_1) in
   let (r_2,θ_2) = decrypt_k'(c_2) in
   encrypt_k'(r_1*r_2,θ_1+θ_2)
  fun re(c) =
   let (r,θ) = decrypt_k'(c) in r*cos(θ)
```

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The Abstraction Property

Untyped contextual equivalence:

`Cartesian` ≡ `Polar`

- **Abstraction holds against any context in the language, even if untyped**

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How to Prove it?

Bisimulations!

- **Conditions for constants, tuples, functions are the same as before**

Bisimulations for Encryption

- Bisimulation respects equality of keys
  - I.e., if k_1 and k_1' are bisimilar, and if k_2 and k_2' are bisimilar, then $k_1 = k_2 \Leftrightarrow k_1' = k_2'$
- For any bisimilar ciphertexts $encrypt_k(v)$ and $encrypt_{k'}(v')$,
  - Neither k nor k' is in the relation, or
  - v and v' are bisimilar

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Example

$$
\begin{aligned}
R = \{\ & (\texttt{Cartesian}, \texttt{Polar}),\\
& (\texttt{Cartesian.make}, \texttt{Polar.make}),\\
& (\texttt{Cartesian.mul}, \texttt{Polar.mul}),\\
& (\texttt{Cartesian.re}, \texttt{Polar.re}),\\
& (\texttt{encrypt}_k(x,y), \texttt{encrypt}_{k'}(r,\theta)),\\
& (z, z)\ |\\
& x = r\cos\theta,\ \ y = r\sin\theta,\ \ z : \texttt{real}\ \}
\end{aligned}
$$

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Formalization

- Defined untyped λ-calculus extended with:
  - Keys **k** and fresh key generation **new x in e**
  - Encryption $\{e_1\}_{e_2}$ and decryption **let** $\{x\}_{e_1} = e_2$ **in** e_3 **else** e_4
    - Assumes perfect encryption
- Proved soundness and completeness of our bisimulations in this language

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Operational Semantics of the Language (1/2)

- Big-step evaluation $(s)e \Downarrow (t)v$ from terms e to values v
  - Annotated with the set of keys s and t before and after the evaluation

$$\frac{k \notin s \qquad (s \cup \{k\})\ [k/x]e \Downarrow (t)v}{(s)\ \text{new } x \text{ in } e \Downarrow (t)v}$$

Operational Semantics of the Language (2/2)

- **Success of decryption:**

$$\frac{(s)e_1 \Downarrow (s_1)k \qquad (s_1)e_2 \Downarrow (s_2)\{v\}_k \qquad (s_2)\ [v/x]e_3 \Downarrow (t)w}{(s)\ \text{let}\ \{x\}_{e_1} = e_2\ \text{in}\ e_3\ \text{else}\ e_4 \Downarrow (t)w}$$

- **Failure of decryption:**

$$\frac{(s)e_1 \Downarrow (s_1)k \qquad (s_1)e_2 \Downarrow (s_2)\{v\}_{k'} \qquad k \neq k' \qquad (s_2)e_4 \Downarrow (t)w}{(s)\ \text{let}\ \{x\}_{e_1} = e_2\ \text{in}\ e_3\ \text{else}\ e_4 \Downarrow (t)w}$$

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Other Examples

- Generative functors
- Non-generative functors
- Encodings of security protocols

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Our Protocol Encoding

- A protocol is encoded as a tuple of participants (and their public keys)
  - Senders are encoded as the values being sent
  - Receivers are encoded as functions from received values to returned values
- Then, contexts play the role of the network, scheduler, and attackers by applying the receivers to the senders

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Example

1. $A \rightarrow B : \{N\}_K$
2. $B \rightarrow A : N \bmod 2$

Sys_N = `new x in`

`(encrypt`$_x$`(N),`

λ`y.(decrypt`$_x$`(y) mod 2))`

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Non-Interference: Secrecy as Equivalence

With our bisimulations, it is easy to prove

$$Sys_M \equiv Sys_N$$

for any M and N with

$$M \bmod 2 = N \bmod 2$$

Which means:

Sys_N keeps N secret (except for its least significant bit) under any context

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Example

$$R = \{ (Sys_M, Sys_N),$$
$$(encrypt_k(M), encrypt_{k'}(N)),$$
$$(\lambda y.(decrypt_k(y) \bmod 2),$$
$$\lambda y.(decrypt_{k'}(y) \bmod 2)),$$
$$(M \bmod 2, N \bmod 2) \}$$

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Protocols Encoded and Proved (or Disproved)

- Needham-Schroeder (insecure)
- Needham-Schroeder-Lowe (secure)
- "ffgg" protocol [Millen 99] (insecure)
  - Attack is "necessarily parallel" but can be simulated in λ-calculus via interleaving

Various properties (such as integrity) can be checked as long as expressed as equivalence

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Needham-Schroeder-Lowe Protocol

1. $\mathsf{B} \rightarrow \mathsf{A} : B$
2. $\mathsf{A} \rightarrow \mathsf{B} : \{N_A, A\}_{k_B}$
3. $\mathsf{B} \rightarrow \mathsf{A} : \{N_A, N_B, B\}_{k_A}$
4. $\mathsf{A} \rightarrow \mathsf{B} : \{N_B\}_{k_B}$
5. $\mathsf{B} \rightarrow \mathsf{A} : \{i\}_{N_B}$

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Encoding of Needham-Schroeder-Lowe Protocol

$$
\begin{aligned}
W &= \langle \lambda x.\, \{x\}_{k_A}, \lambda x.\, \{x\}_{k_B}, k_E, U, V \rangle \\
U &= \langle B, \lambda \{\langle x, y \rangle\}_{k_B}.\, \mathtt{assert}(y = A); \\
&\qquad \nu z.\, \langle \{\langle x, z, B \rangle\}_{k_A}, \\
&\qquad\quad \lambda \{z_0\}_{k_B}.\, \mathtt{assert}(z_0 = z); \\
&\qquad\qquad \{i\}_z \rangle \rangle \\
V &= \lambda x.\, \mathtt{let}\ k_x = (\mathtt{if}\ x = B\ \mathtt{then}\ k_B\ \mathtt{else} \\
&\qquad\qquad \mathtt{if}\ x = E\ \mathtt{then}\ k_E\ \mathtt{else}\ \bot)\ \mathtt{in} \\
&\qquad \nu y.\, \langle \{(y, A)\}_{k_x}, \\
&\qquad\quad \lambda \{\langle y_0, z, x_0 \rangle\}_{k_A}.\, \mathtt{assert}(y_0 = y); \\
&\qquad\qquad \mathtt{assert}(x_0 = x); \\
&\qquad\qquad \{z\}_{k_x} \rangle
\end{aligned}
$$

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Bisimulation for Needham-Schroeder-Lowe Protocol

$$
\begin{array}{l}
\{(\mathsf{U},\mathsf{U}'),(\mathsf{V},\mathsf{V}'),(\mathsf{W},\mathsf{W}'),\\
(\overline{k},\overline{k}'),(A,A),(B,B),(E,E),(\lambda x.\,\{x\}_{k_A},\lambda x.\,\{x\}_{k'_A}),(\lambda x.\,\{x\}_{k_B},\lambda x.\,\{x\}_{k'_B}),(k_E,k'_E),(\overline{w},\overline{w}'),(\{\overline{w}\}_{k_A},\{\overline{w}'\}_{k'_A}),(\{\overline{w}\}_{k_B},\{\overline{w}'\}_{k'_B}),\\
(\lambda\{\langle x,y\rangle\}_{k_B}.\,\mathtt{assert}(y=A);\nu z.\,(\{\langle x,z,B\rangle\}_{k_A},\lambda\{z_0\}_{k_B}.\,\mathtt{assert}(z_0=z);\{i\}_z),\ \lambda\{\langle x,y\rangle\}_{k'_B}.\,\mathtt{assert}(y=A);\nu z.\,(\{\langle x,z,B\rangle\}_{k'_B},\lambda\{z_0\}_{k'_B}.\,\mathtt{assert}(z_0=z);\{j\}_z)),\\
(\langle\{\{\langle\overline{k}_{AB},A\rangle\}_{k_B},\lambda\{\langle y_0,z,x_0\rangle\}_{k_A}.\,\mathtt{assert}(y_0=\overline{k}_{AB});\mathtt{assert}(x_0=B);\{z\}_{k_B}\rangle,\ \langle\{\{\langle\overline{k}'_{AB},A\rangle\}_{k'_B},\lambda\{\langle y_0,z,x_0\rangle\}_{k'_A}.\,\mathtt{assert}(y_0=\overline{k}'_{AB});\mathtt{assert}(x_0=B);\{z\}_{k'_B}\rangle),\\
(\{\{\langle\overline{k}_{AB},A\rangle\}_{k_B},\{\{\langle\overline{k}'_{AB},A\rangle\}_{k'_B}),\\
(\lambda\{\langle y_0,z,x_0\rangle\}_{k_A}.\,\mathtt{assert}(y_0=\overline{k}_{AB});\mathtt{assert}(x_0=B);\{z\}_{k_B},\ \lambda\{\langle y_0,z,x_0\rangle\}_{k'_A}.\,\mathtt{assert}(y_0=\overline{k}'_{AB});\mathtt{assert}(x_0=B);\{z\}_{k'_B}),\\
(\langle\{\{\langle\overline{k}_{AB},\overline{k}_B,B\rangle\}_{k_A},\lambda\{z_0\}_{k_B}.\,\mathtt{assert}(z_0=\overline{k}_{AB});\{i\}_{\overline{k}_B}\rangle,\ \langle\{\{\langle\overline{k}'_{AB},\overline{k}'_B,B\rangle\}_{k'_A},\lambda\{z_0\}_{k'_B}.\,\mathtt{assert}(z_0=\overline{k}'_{AB});\{j\}_{\overline{k}'_B}\rangle),\\
(\{\{\langle\overline{k}_{AB},\overline{k}_B,B\rangle\}_{k_A},\{\{\langle\overline{k}'_{AB},\overline{k}'_B,B\rangle\}_{k'_A}),\\
(\lambda\{z_0\}_{k_B}.\,\mathtt{assert}(z_0=\overline{k}_{AB});\{i\}_{\overline{k}_B},\ \lambda\{z_0\}_{k'_B}.\,\mathtt{assert}(z_0=\overline{k}'_{AB});\{j\}_{\overline{k}'_B}),\\
(\{\overline{k}_B\}_{k_B},\{\overline{k}'_B\}_{k'_B}),\\
(\{i\}_{\overline{k}_B},\{j\}_{\overline{k}'_B}),\\
(\langle\{\{\langle\overline{k}_{AE},A\rangle\}_{k_B},\lambda\{\langle y_0,z,x_0\rangle\}_{k_A}.\,\mathtt{assert}(y_0=\overline{k}_{AE});\mathtt{assert}(x_0=E);\{z\}_{k_E}\rangle,\ \langle\{\{\langle\overline{k}'_{AE},A\rangle\}_{k'_E},\lambda\{\langle y_0,z,x_0\rangle\}_{k'_A}.\,\mathtt{assert}(y_0=\overline{k}'_{AE});\mathtt{assert}(x_0=E);\{z\}_{k'_B}\rangle),\\
(\{\{\langle\overline{k}_{AE},A\rangle\}_{k_B},\{\{\langle\overline{k}'_{AE},A\rangle\}_{k'_E}),\\
(\lambda\{\langle y_0,z,x_0\rangle\}_{k_A}.\,\mathtt{assert}(y_0=\overline{k}_{AE});\mathtt{assert}(x_0=E);\{z\}_{k_E},\ \lambda\{\langle y_0,z,x_0\rangle\}_{k'_A}.\,\mathtt{assert}(y_0=\overline{k}'_{AE});\mathtt{assert}(x_0=E);\{z\}_{k'_E}),\\
(\langle\overline{k}_{AE},A\rangle,\langle\overline{k}'_{AE},A\rangle),\\
(\overline{k}_{AE},\overline{k}'_{AE}),\\
(\langle\{\{\langle\overline{w},\overline{k}_B,B\rangle\}_{k_A},\lambda\{z_0\}_{k_B}.\,\mathtt{assert}(z_0=\overline{k}_B);\{i\}_{\overline{k}_B}\rangle,\ \langle\{\{\langle\overline{w}',\overline{k}'_B,B\rangle\}_{k'_A},\lambda\{z_0\}_{k'_B}.\,\mathtt{assert}(z_0=\overline{k}'_B);\{j\}_{\overline{k}'_B}\rangle),\\
(\{\{\langle\overline{w},\overline{k}_B,B\rangle\}_{k_A},\{\{\langle\overline{w}',\overline{k}'_B,B\rangle\}_{k'_A}),\\
(\lambda\{z_0\}_{k_B}.\,\mathtt{assert}(z_0=\overline{k}_B);\{i\}_{\overline{k}_B},\ \lambda\{z_0\}_{k'_B}.\,\mathtt{assert}(z_0=\overline{k}'_B);\{j\}_{\overline{k}'_B})),\\
(\{\overline{w}\}_{k_B},\{\overline{w}'\}_{k'_B})\}
\end{array}
$$

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Related Work (1/5): Logical Relations

- **Semantic logical relations**
  [Tait 67, Plotkin 73, Reynolds 83, Mitchell 93, etc.]
  - **Suffers from the complexity and imprecision of denotational semantics for recursion**
- **Syntactic logical relations**
  [Pitts 98, Birkedal-Harper 97, etc.]
  - **Still suffers from complications for recursive functions/types**

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Related Work (2/5): Applicative Bisimulations

- For untyped λ-calculus [Abramsky 90]
- For object calculi with universal and subtyping polymorphism [Gordon-Rees]

None can deal with type abstraction (i.e., existential polymorphism)

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Related Work (3/5): Bisimulations for π-Calculi

- For polymorphic π-calculus
  [Pierce-Sangiorgi 97, Berger-Honda-Yoshida 03]
- For spi-calculus
  [Abadi-Gordon 98, Boreale-DeNicola-Pugliese 99, Borgstrom-Nestmann 02, Abadi-Fournet 01, etc.]

Incomplete, or completeness claimed but proof unpublished (or found wrong)

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Related Work (4/5): Operational Models of Types

- **Indexed models** [Appel et al.]
- **Operational ideal models** [Voillon-Melliès 04, etc.]

Only unary case (safety) considered;
Binary case (equivalence) left open

- **Non-trivial in the presence of existential types**

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Related Work (5/5)

- **Categorical reformulation of our logical relations for perfect encryption**
  [Goubault-Larrecq-Lasota-Nowak-Zhang, CSL 04]

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Future Directions (1/3)

- Translations between different forms of information hiding
  - E.g. from type abstraction to encryption
  - Challenge: How to prove full abstraction (preservation of equivalence)?
- Extension to even more expressive languages (e.g. higher-order π-calculus)

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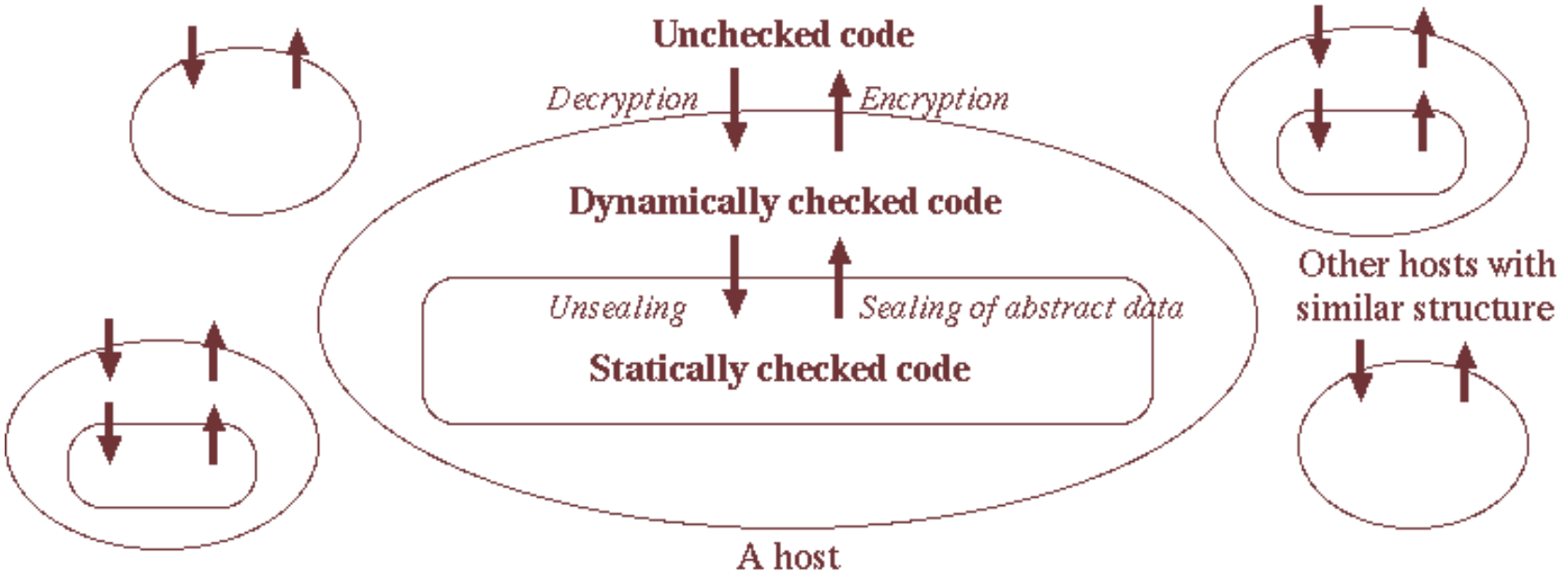
Future Directions (2/3)

- Adoptation/generalization for other forms of information hiding (e.g. security typing)
  - Generative security levels?
- A system allowing unchecked, dynamically checked, and statically typed code without losing abstraction
  - C, Perl, ML, etc. coexist in peace?

Future Directions (3/3)



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